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Mathematical modelling of filtration processes in drainage systems with different depths of drainage

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Abstract: The article discusses the option for the application of the methodology for the solution of boundary value problems on the conformal mapping for the calculation of filtration process in the horizontal systematic drainage, provided that the drain is installed at a different depth. In particular, the case of methods combining fictitious areas and quasiconformal mappings for solving nonlinear boundary conditions problems for calculating filtration regimes in soils with free sections of boundaries (depression curves) and intervals of the "drainage" type.

As an example, the authors designed a hydrodynamic flow grid, determined the values of the flows to the drain, established a section line and elicited other process characteristics. The algorithm for the numerical solution of model nonlinear boundary conditions problems of quasiconformal reflection in areas bounded by two equipotential lines and two flow lines, when for one of the sections, the boundary is an unknown (free) curve with fixed and free ends.

The conducted numerical calculations prove that the problems and algorithms of their numerical solution, with a relatively small iterations number (k = 141) suggested in the paper, can be applied in the simulation of nonlinear filtration processes that arise in horizontal drainage systems. Total filtration flow obtained Q = 0.9 $dm^3 \cdot s^{-1}$; flow for drains Q_1 = 0.55 $dm^3 \cdot s^{-1}$ and Q_2 = 0.35 $dm^3 \cdot s^{-1}$ are quite consistent with practically determined values.

Keywords: conformal mapping, drainage systems, filtration processes, flood protection, mathematical modelling

INTRODUCTION

In order to create necessary water conditions for agricultural production and to protect areas and settlements from surface water and groundwater flooding as a result of their dynamics (water level rise and fall), there is a need to solve a number of issues in the first place, the regulation of groundwater levels with the use of drainage systems, in particular, the placement of regulating drains at different depths (Fig. 1) [Castro-Orgaz, Hager 2017; Rokochinskiy *et al.* 2019a; Sato, Iwasa 2000; Smedema 2011; Tkachuk *et al.* 2014; van der Molen *et al.* 2007]. This problem is especially important for reclamation, where regulating drains are the major element of any drainage system [Klimov *et al.* 2019; Rokochinskiy *et al.* 2019b; Tkachuk *et al.* 2015].

Currently, numerical studies of the corresponding boundary value problems confirm that the method of inverse boundary value problems (conformal and quasiconformal mapping) is the most effective one. Specifically, Bomba *et al.* [2007] examined the case of combining the fictitious domain method and quasiconformal mapping used for the solution of nonlinear boundary value problems in order to calculate filtration regimes in media with free boundary areas (depression curves) and "outflow" type spaces. Additionally, Bomba *et al.* [2008] suggested an algorithm for a numerical solution of model nonlinear boundary value problems on the quasiconformal maps in the areas bounded by two equipotential lines and two-course (flow) lines, when one of the boundary parts is an unknown (free) curve with fixed and free ends. Polubarinova-Kochina [1948] considered the stationary problem of flat-vertical stationary non-pressure stationary liquid

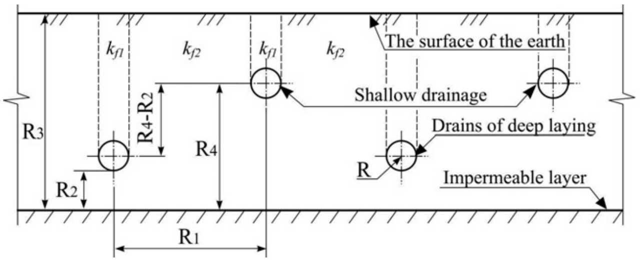

Fig. 1. Drainage systems with different depths of drainage: R = radius of the regulating drain; R_1 = distance between the regulating drains; R_2 = the distance from the deep laying drainage to the impermeable layer; R_3 = the distance from the earth's surface to the impermeable layer; R_4 = the distance from the shallow drainage to the impermeable layer; $R_4 - R_2$ = the distance between the deep drainage level and shallow drainage level; k_{f1}, k_{f2} = filtration coefficient drainage backfill and the main (unloose) soil mass; source: own elaboration

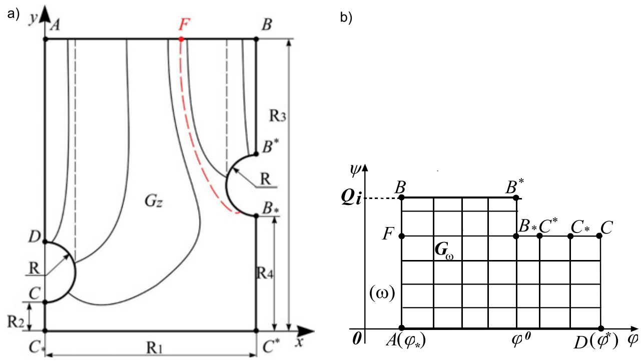

Fig. 2. Graphic interpretation of the filtration process: a) filtration area G_z, b) the corresponding area of complex potential G_ω; symbols used in the Fig. are explained in the text in p. 75–76; source: own elaboration

filtration to horizontal symmetric drainage when there is free groundwater surface (depression curve).

In this paper, we introduce the application of a conformal mapping methodology for solving boundary value problems in order to calculate the filtration process in a horizontal drain, provided that the drains are installed at a different depth. Depending on the difference in the drain depth, there are different options of flow formation. For instance, the cases start with the complete flow symmetry, when the drains are installed at the same depth, then move along with a gradual shift of the boundary line towards the drains with a lower depth of installation (Fig. 2), and end with the case when the flow to the drain tends to zero.

MATERIALS AND METHODS

STATEMENT OF THE PROBLEM

Let us consider the filtration process for the horizontal drainage under the conditions of installing a series of drain on two minor different depths. Considering the symmetry of the motion pattern, the research provides insight only into one fragment of such a system, namely, the curvilinear area $G_z = ABB^*B_*C^*C_*CD$ ($z = x + iy$) (Fig. 2) limited by the flow lines $BB^* = \{z: x = R_1, R_4 + 2R \le y \le R_3\}$, $B_*C^* = \{z: x = R_1, 0 \le y \le R_4\}$, $C^*C_* = \{z: y = 0, 0 \le x \le R_1\}$, $C_*C = \{z: x = 0, 0 \le y \le R_2\}$, $DA = \{z: x = 0, R_2 + 2R \le \le y \le R_3\}$ and equipotential lines $B^*B_* = \{z: (x - R_1)^2 + (y - R_4 - R)^2 = R^2, R_4 \le y \le R_4 + 2R\}$, $AB = \{z: y = R_3, 0 \le x \le R_1\}$, $CD = \{z: x^2 + (y - R_2 - R)^2 = R^2, R_2 \le y \le R_2 + 2R\}$.

Similarly to other researchers, like POLUBARINOVA-KOCHINA [1948], SAMARSKIY [1977], MARCHUK [1989] and BOMBA *et al.* [2007; 2008; 2018], we shall describe the noted fragment with the equation of motion $\vec{\upsilon} = k_f \mathrm{grad}\varphi$ (Darcy's law) and the continuity equation $\mathrm{div}\vec{\upsilon} = 0$, where $\vec{\upsilon} = \big(\upsilon_x(x,y) + i\cdot\upsilon_y(x,y)\big)$ is filtration velocity, k_f is filtration coefficient, bearing in mind that $k_f = k_{f1}$, if $0 \le x < R$, $R_2 + R < y \le R_3$, $R_1 - R \le x < R_1$, otherwise $k_f = k_{f2}$, $\varphi = \varphi(x, y)$ is 2–3 the potential at the point (x, y).

Taking into account the research results revealed in BOMBA *et al.* [2007; 2008; 2018], putting a harmonic function $\psi = \psi(x, y)$ (flow function), complex conjugate to $\varphi = \varphi(x, y)$, and by the adoption of a number of boundary conditions, form a more general the corresponding conformal mapping problem $\omega = \omega(z) = \varphi(x, y) + i\psi(x, y)$ of the considered area G_z to the corresponding complex potential area $G_\omega = \{\omega: \varphi_* < \varphi < \varphi^*, 0 < \psi < Q\}$ ($Q = Q_1 + Q_2$ – initially unknown total filtration flow rate) looks as follows:

$$k_f\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y},\; k_f\frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x}; \tag{1}$$

$$\begin{gathered}\varphi|_{AB} = \varphi_*,\; \varphi|_{CD} = \varphi^*,\; \varphi|_{B^*B_*} = \varphi^0,\\ \psi|_{AD} = 0,\; \psi|_{BB^*} = Q_2,\; \psi|_{B_*C^*C_*C} = Q_1.\end{gathered} \tag{2}$$

We will formulate the conformal mapping problem $z = z(\omega) = x(\varphi, \psi) + iy(\varphi, \psi)$ of the area G_ω on G_z, which is inverse to Equations (1)–(2), for unknown values of Q_1 and Q_2 in the following form:

$$\begin{cases} k_f\frac{\partial y}{\partial\psi} = \frac{\partial x}{\partial\varphi},\\ k_f\frac{\partial x}{\partial\psi} = -\frac{\partial y}{\partial\varphi},\end{cases} \quad (\varphi,\psi)\in G_\omega \tag{3}$$

$$\begin{cases} y(\varphi_*,\psi) = R_3, 0 \le x \le R_1, & 0 \le \psi \le Q,\\ x(\varphi^*,\psi)^2 + (y(\varphi^*,\psi) - R_2 - R)^2 = R^2, R_2 \le y \le R_2 + 2R, & 0 \le \psi \le Q_1,\\ \left(x(\varphi^0,\psi) - R_1\right)^2 + \left(y(\varphi^0,\psi) - R_4 - R\right)^2 = R^2, R_4 \le y \le R_4 + 2R, & Q_1 \le \psi \le Q_2,\\ x(\varphi, Q_1) = R_1, 0 \le y \le R_4, & \varphi^0 \le \varphi \le \varphi^*,\\ x(\varphi, Q_2) = R_1, R_4 + 2R \le y \le R_3, & 0 \le \varphi \le \varphi^0,\\ y(\varphi, Q_1) = 0, 0 \le x \le R_1, & \varphi^0 \le \varphi \le \varphi^*,\\ x(\varphi, Q_1) = 0, 0 \le y \le R_2, & \varphi^0 \le \varphi \le \varphi^*,\\ x(\varphi, 0) = 0, R_2 + 2R \le y \le R_3, & \varphi_* \le \varphi \le \varphi^*,\end{cases} \tag{4}$$

The transition from direct (Eqs. 1, 2) to inverse (Eqs. 3, 4) tasks has a number of advantages: the inverse task has a rectangular grid G_ω (the direct task has a curvilinear grid G_z), which makes a hydrodynamic grid of motion, i.e. simultaneously with the construction of such a hydrodynamic grid of motion is determined by the total filtration flow and its components

$Q = Q_1 + Q_2$, in this case, the nonlinearity generated by the transition from the direct to the inverse task is localised and does not significantly affect the problem calculation.

The corresponding second-order equations in order to find the functions $x = x(\varphi, \psi)$ and $y = y(\varphi, \psi)$ in a divergent form are as follows:

$$\begin{gathered}\frac{\partial}{\partial \varphi}\left(\frac{1}{k_f}\frac{\partial x}{\partial \varphi}\right)+\frac{\partial}{\partial \psi}\left(k_f\frac{\partial x}{\partial \psi}\right)=0\\ \frac{\partial}{\partial \varphi}\left(\frac{1}{k_f}\frac{\partial y}{\partial \varphi}\right)+\frac{\partial}{\partial \psi}\left(k_f\frac{\partial y}{\partial \psi}\right)=0\end{gathered} \tag{5}$$

ALGORITHM FOR THE NUMERICAL SOLUTION OF THE PROBLEM

Numerical solutions to such problems were considered in the works of BOHAIENKO [2019], DIETHELM [2011], BERESLAVSKII [2014], GUBKINA *et al.* [2007]. To numerically plot the mapping of the rectangle G_ω on a curved quadrangular area G_z (and comparing the corresponding points A, B, B*, B* etc. – see Fig. 2), following the method described by MARCHUK [1989], the difference analogue of Equations (5), boundary conditions (4) and boundary orthogonality conditions in the corresponding uniform grid domain shall be written
$G_\omega^\gamma = \{(\varphi_i, \psi_j);\ \varphi_i = \Delta\varphi \cdot i,\ i = \overline{0, m+1};$
$\psi_j = \Delta\psi \cdot j,\ j = \overline{0, n+1};\ \Delta\psi = Q/(n+1),\ \Delta\psi = Q/(n+1),$
$\gamma = \Delta\varphi/\Delta\psi, \quad m,\ n \in \mathbf{N}\}$ (where $\Delta\varphi$ and $\Delta\psi$ – grid steps respectively by and ψ) as follows:

$$\begin{cases}\sigma\left(a_{i+1,j+1}x_{i+1,j+1}-\left(a_{i+1,j+1}+a_{i,j+1}\right)x_{i,j+1}+a_{i,j+1}x_{i-1,j+1}\right)+\\ +(1-2\sigma)\left(a_{i+1,j}x_{i+1,j}-\left(a_{i+1,j}+a_{i,j}\right)x_{i,j}+a_{i,j}x_{i-1,j}\right)+\\ +\sigma\left(a_{i+1,j-1}x_{i+1,j-1}-\left(a_{i+1,j-1}+a_{i,j-1}\right)x_{i,j-1}+a_{i,j-1}x_{i-1,j-1}\right)+\\ +\gamma^2\left(\sigma\left(b_{i+1,j+1}x_{i+1,j+1}-\left(b_{i+1,j+1}+b_{i+1,j}\right)x_{i+1,j}+b_{i+1,j}x_{i+1,j-1}\right)+\right.\\ +(1-2\sigma)\left(b_{i,j+1}x_{i,j+1}-\left(b_{i,j+1}+b_{i,j}\right)x_{i,j}+b_{i,j}x_{i,j-1}\right)+\\ \left.+\sigma\left(b_{i-1,j+1}x_{i-1,j+1}-\left(b_{i-1,j+1}+b_{i-1,j}\right)x_{i-1,j}+b_{i-1,j}x_{i-1,j-1}\right)\right)=0,\\ \sigma\left(a_{i+1,j+1}y_{i+1,j+1}-\left(a_{i+1,j+1}+a_{i,j+1}\right)y_{i,j+1}+a_{i,j+1}y_{i-1,j+1}\right)+\\ +(1-2\sigma)\left(a_{i+1,j}y_{i+1,j}-\left(a_{i+1,j}+a_{i,j}\right)y_{i,j}+a_{i,j}y_{i-1,j}\right)+\\ +\sigma\left(a_{i+1,j-1}y_{i+1,j-1}-\left(a_{i+1,j-1}+a_{i,j-1}\right)y_{i,j-1}+a_{i,j-1}y_{i-1,j-1}\right)+\\ +\gamma^2\left(\sigma\left(b_{i+1,j+1}y_{i+1,j+1}-\left(b_{i+1,j+1}+b_{i+1,j}\right)y_{i+1,j}+b_{i+1,j}y_{i+1,j-1}\right)+\right.\\ +(1-2\sigma)\left(b_{i,j+1}y_{i,j+1}-\left(b_{i,j+1}+b_{i,j}\right)y_{i,j}+b_{i,j}y_{i,j-1}\right)+\\ \left.+\sigma\left(b_{i-1,j+1}y_{i-1,j+1}-\left(b_{i-1,j+1}+b_{i-1,j}\right)y_{i-1,j}+b_{i-1,j}y_{i-1,j-1}\right)\right)=0,\\ \qquad i=\overline{1,m}, \quad j=\overline{1,n}.\end{cases} \tag{6}$$

where:
$a_{i,j} = \frac{2}{k_{f_{i,j}}+k_{f_{i-1,j}}}, b_{i,j} = \frac{k_{f_{i,j-1}}+k_{f_{i,j}}}{2}, k_{f_{i+\frac{1}{2}j+\frac{1}{2}}} = \frac{k_{f_{ij}}+k_{f_{i+1j}}+k_{f_{ij+1}}+k_{f_{i+1j+1}}}{4}$,
$x_{i,j} = x(\varphi_i, \psi_j), y_{i,j} = y(\varphi_i, \psi_j), k_{fi,j} = k_f(\varphi_i, \psi_j)$, σ [0, 0.5] is weighting factor, which affects the stability, accuracy, and rate of convergence of the difference scheme.

Boundary conditions – functions that determine the physical area G_z – approximated by point-difference equations for x and y, which include boundary nodes:

$$\begin{cases}y_{0j}=R_3, \quad j=\overline{0,n+1},\\ x_{i0}=0, \qquad i=\overline{1,m+1},\\ x_{in_0}=0, \quad i=\overline{m-m_1+1,m},\\ y_{in_0}=0, \quad i=\overline{m-m_1-m_2,m-m_1},\\ x_{in+1}=R_1, \quad i=\overline{0,m_0-1},\\ \left(x_{m_0j}-R_1\right)^2+\left(y_{m_0j}-R_4-R\right)^2=R^2, \quad j=\overline{n_0+1,n+1},\\ x^2{}_{m+1j}+\left(y_{m+1j}-R_2-R\right)^2=R^2, \quad j=\overline{0,n_0},\\ x_{in_0}=R_1, \quad i=\overline{m_0,m-m_1-m_2-1}.\end{cases} \tag{7}$$

$$\begin{cases}-x_{1j}+x_{0j}=0, \quad j=\overline{0,n}+1,\\ y_{in}-y_{in+1}=0, \quad i=\overline{0,m_0-1},\\ -x_{in0}+x_{in_0-1}=0, \quad i=\overline{m-m_1-m_2,m-m_1},\\ y_{in_0}-y_{in_0-1}=0, \quad i=\overline{m-m_1+1,m},\\ y_{in_0}-y_{in_0-1}=0, \quad i=\overline{m_0,m-m_1-m_2-1},\\ 2x_{m+1j}\left(y_{mj}-y_{m+1j}\right)-2\left(y_{m+1j}-R_2-R\right)\left(x_{mj}-x_{m+1j}\right)=0,\\ \qquad j=\overline{0,n_0},\\ 2\left(x_{m_0j}-R_1\right)\left(y_{m_0-1j}-y_{m_0j}\right)-2\left(y_{m_0j}-R_4-R\right)\left(x_{m_0-1j}-x_{m_0j}\right)=0,\\ \qquad j=\overline{n_0+1,n+1},\\ y_{i1}-y_{i0}=0, \quad i=\overline{1,m.}\end{cases} \tag{8}$$

Conformal invariant γ of a curvilinear quadrilateral G_z (the ratio of the rectangle sides G_ω) is unknown because the cost is unknown before) and is determined in the calculation process. The equation for the approximate calculating of this value is obtained on the basis of the condition of "conformal similarity in small" of the corresponding quadrilaterals (rectangles) of two areas

$$\gamma = \frac{1}{(m+1)(n+1)}\sum\nolimits_{i,j=0}^{m,n}\frac{1}{k_{f_{i+\frac{1}{2},j+\frac{1}{2}}}}\gamma_{i,j} \tag{9}$$

where:

$$\gamma_{i,j} = \frac{\sqrt{\left(x_{i+1,j}-x_{i,j}\right)^2+\left(y_{i+1,j}-y_{i,j}\right)^2}+\sqrt{\left(x_{i+1,j+1}-x_{i,j+1}\right)^2+\left(y_{i+1,j+1}-y_{i,j+1}\right)^2}}{\sqrt{\left(x_{i,j+1}-x_{i,j}\right)^2+\left(y_{i,j+1}-y_{i,j}\right)^2}+\sqrt{\left(x_{i+1,j+1}-x_{i+1,j}\right)^2+\left(y_{i+1,j+1}-y_{i+1,j}\right)^2}}$$

We shall form the solution of the difference problem (Eq. (6) – Eq. (9)) following the schemes suggested by BOMBA *et al.* [2007; 2008; 2018]. We assign the quantities of m and n nodes partitioning the grid domain G_ω, parameter ε, which characterises the approximation accuracy of the solution of the corresponding difference problem. We set the initial approximation of a series of variables: the initial approximation of the coordinates of the boundary nodes $x_{0,j}^{(0)}$, $y_{0,j}^{(0)}$, $x_{m+1,j}^{(0)}$, $y_{m+1,j}^{(0)}$, $x_{i,n+1}^{(0)}$, $y_{i,n+1}^{(0)}$, $x_{i,0}^{(0)}$, $y_{i,0}^{(0)}$ (so that the Eqs. (7) are fulfilled) and the initial approximation of the coordinates of the internal nodes $\left(x_{i,j}^{(0)},\ y_{i,j}^{(0)}\right)$, $i = \overline{1,\ m}$, $j = \overline{1,\ n}$. We will use the equation (9) in order to define the initial approximation of a conformal invariant γ, moreover, in this case, we will use the newly given initial values of the internal nodes' coordinates, i.e. $\gamma^{(0)} = \gamma\left(x_{i,j}^{(0)}, y_{i,j}^{(0)}\right)$. Then, we will refine the following parameters:

- internal nodes $\left(x_{i,j}{}^{(k+1)},\ y_{i,j}{}^{(k+1)}\right)$ (k = 0, 1, … is the number of the iteration step) using the Gauss–Seidel method presented by SAMARSKIY [1977] according to the equations obtained by solving Equations (6) with respect to $x_{i,j}$ and $y_{i,j}$;
- γ values according to the equation (9) and value of filtration water flow rate Q according to the equation $Q = \frac{1}{\gamma}\,\frac{n+1}{m+1}$;
- coordinates of boundary nodes, for example, by solving a system of nonlinear equations (7), (8).

Next, we verify the fulfilment of the conditions for the termination of the computational process, for example, according to the following equations:

$$\begin{gathered}\max_{x_{i,j},\, y_{i,j} \in \partial G_z}\left(\left|x_{i,j}{}^{(k+1)}-x_{i,j}{}^{(k)}\right|,\ \left|y_{i,j}{}^{(k+1)}-y_{i,j}{}^{(k)}\right|\right)<\varepsilon,\\ \left|Q^{(k+1)}-Q^{(k)}\right|<\varepsilon, \left|D^{(k+1)}-D^{(k)}\right|<\varepsilon,\end{gathered} \tag{10}$$

where: $D = \frac{1}{(m+1)(n+1)}\sum\nolimits_{i,j=0}^{m,n}\frac{\sqrt{\left(x_{i+1,j+1}-x_{i,j}\right)^2+\left(y_{i+1,j+1}-y_{i,j}\right)^2}}{\sqrt{\left(x_{i,j+1}-x_{i+1,j}\right)^2+\left(y_{i,j+1}-y_{i+1,j}\right)^2}}$

is the average value of the ratio of the lengths of the curvilinear quadrilaterals' diagonals within the grid domain.

If the conditions of equation (10) are not fulfilled, we return to the refinement of the internal nodes' coordinates, etc. Otherwise, we calculate the deviation of the conformity of the resulting grid according to the Equation $\varepsilon_* = |1 - D|$. Its value characterises the deviation of the resulting curvilinear quadrilaterals from the corresponding rectangles (since the ratio of the diagonals' lengths in a rectangle is equal to one, whereas the existence of right angles is provided by orthogonality conditions).

If, for instance, only one of the conditions of equation (10) is not fulfilled, we reconcile the ratio of accuracy ε_* and the given number of partitioning steps m, n (first of all, by increasing the latter ones). However, there is a need to increase the degree of accuracy of the approximate solution (to reduce the deviation ε_*), we increase the partition parameters m and n and solve the difference problem (Eq. (6) – Eq. (9)) again. Similarly, to Bomba *et al.* [2007; 2008; 2018], we achieve the optimality of the relation between m and n by optimising the analogues of functional equations (for example, Riemann's functional equation). The rationale for the constructed algorithm for "alternating fixation of the process and environment characteristics, the conformal parameter, internal and boundary nodes of the curvilinear area" is the same as in papers by Bomba *et al.* [2007; 2008; 2018], with the use of ideas of block iterative methods described by Samarskiy [1977].

RESULTS AND DISCUSSION

The proposed methodology for solving boundary value tasks on conformal mappings can apply for calculating filtration processes occurring within modular drainage systems (innovative spatial systems with different runoff depths), provided that the regulating drains are laid at different depths (Fig. 1). Wherein, as an example, the solution of the task was given. The corresponding numerical calculations were carried out, and the hydrodynamic grid was plotted (Fig. 3). A line for changing the filtration flow directions was established, determining the values of flows in the regulating drains located at different depths and other characteristics of filtration processes.

As a result of computation under the described algorithm using the values of the following parameters: R = 0.05 m; R_1 = 20 m; R_2 = 1.0 m; R_4 = 1.8 m; R_3 = 3.0 m; $R_4 - R_3$ = 0.8 m; k_{f1} = 1.2 m·d^{-1}; k_{f2} = 0.8 m·d^{-1} – the average permeability of the sandy loam in the loose condition and the condition of natural occurrence, respectively [Kozlowski, Ludynia 2019] (see designations Fig. 1a) m = 30 m, n = 30 – the number of split nodes of the grid area; m_1 = 0; m_2 = 7; m_3 = 3; $\varphi_* = 0$, $\varphi^0 = R_3 - R - R_4$; $\varphi_* = R_3 - R - R_2$ (boundary conditions, see Fig. 1b); ε = 0.0001 were found: total filtration flow Q = 0.9 dm^3·s^{-1}; flow for drains Q_1 = 0.55 dm^3·s^{-1} and Q_2 = 0.35 dm^3·s^{-1} by k = 141 iterations, furthermore, we obtained a hydrodynamic flow grid (Fig. 3). In particular, in the Table 1 shows the dependences of drainage flow Q_1 and Q_2 from the changes of the drainage occurrence depth R_4 and R_2 (in this example parameters R_4 = 1.8 m, R_2 = 1.0 m).

The hydrodynamic grid (lines of flow and lines of equal pressure, Fig. 3) makes it possible to calculate the flow of water to the regulating drains located at different depths, and also to calculate the most important parameters for calculating these drains – the distance between them.

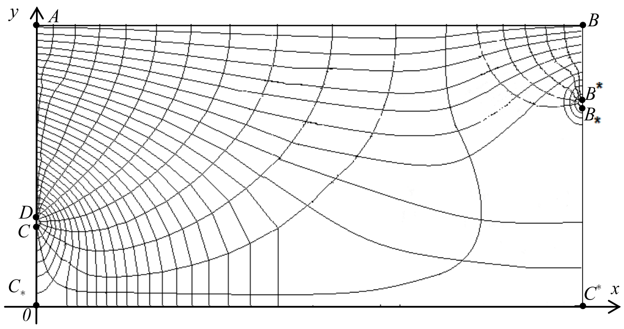

Fig. 3. Hydrodynamic flow grid in the physical area G_z; source: own study

Table 1. Dependences of drainage flow Q_1 and Q_2 from the changes of the drainage occurrence depth R_4 and the distance from the deep laying drainage to the impermeable layer R_2 =1.0 m

Q_1 (dm^3·s^{-1})	R_4 (m)	Q_2 (dm^3·s^{-1})
0.550	1.8	0.350
0.557	1.9	0.342
0.564	2.0	0.337
0.572	2.1	0.330
0.577	2.2	0.323
0.585	2.25	0.317
0.590	2.3	0.302

Source: own study.

CONCLUSIONS

The paper introduces the application of a conformal mapping methodology for solving boundary value problems in order to calculate the filtration process in a horizontal drain, provided that the drains are installed at a different depth. Concurrently, the researchers constructed an algorithm for the numerical solution of the problem, conducted corresponding numerical calculations, created a hydrodynamic flow grid, established a flow partition line, found the values of flows (migration) to drains, and other characteristics of the process.

The conducted numerical calculations prove that the problems and algorithms of their numerical solution suggested in the paper can be applied in the simulation of nonlinear filtration processes that arise in horizontal drainage systems, as well as in the design of drainage facilities.

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