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Cross-waves and pulsating flows in the side-channel spillway – an experimental approach

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Abstract: Potentially hazardous side-channels of complex geometry need to be investigated using detailed hydraulic physical models. This study aims to analyse the cross-waves pattern and pulsating flow using a side-channel spillway physical model. This study compares the cross-waves pattern were measured using an experimental installation set to generate cross-waves on the surface (original series) with another structure that did not produce cross-waves (modified series). The results showed that the geometry of the left wall caused instability in flow patterns and secondary flows. The starting point of Q_2 discharge was detected by minor turbulence on the water surface near the left wall at a water depth of 3.3 m at the starting point of the wall, but with no overtopping. Cross-waves formed downstream at the right wall crosswise, lower than at the left wall. The height of the cross-wave increased substantially from Q_{100} to Q_{1000} discharges leading to overtoppings near the left wall at a water depths of 4.2 and 5.0 m at the starting point of the wall, and near the right wall at a water depths of 3.8 and 4.0 m at the upstream point of the wall. The modifications provided optimal hydraulic conditions, i.e. elimination of cross-waves and non-uniform flows. The Vedernikov and Montouri numbers showed that both original and modified series did not enter the area where the pulsating flow occurred. This indicated that both series were free from the pulsating flow.

Keywords: cross-waves, hydrodynamic force, physical model, pulsating flow, side-channel spillway

INTRODUCTION

The building of dams or reservoirs to accumulate water resources helps to overcome water scarcity. A dam is a water storage structure that does not permit overtopping. It is equipped with a spillway to avoid its collapse due to large hydrostatic pressure caused by overflows [Kumcu 2016]. Dams cause water levels to rise, which leads to an energy (head) difference between the upstream and downstream of the spillway. The upstream water that passes through the spillway has a large amount of energy that causes greater flow velocity. After passing the spillway, the flow is in a supercritical state, while it becomes subcritical at the downstream slope.

A side-channel is an energy dissipation structure with a trapezoid or rectangular cross-section located at the end of the control channel and equipped with a regulating weir. The energy dissipation process causes the channel to receive hydrodynamic forces as flow impacts and vibration forces. A detailed hydraulic physical model should be used to study side-channels of complex geometry and major hazard potentials [Azmeri *et al.* 2021; Lucas *et al.* 2015; Yulianur *et al.* 2022]. The deflection in the side-channel structure triggers cross-waves at the downstream transition channel. Therefore, installing sills at the beginning of the deflection structure is expected to homogenise the flow direction and prevent cross-waves [Akmal 2014; Lopes *et al.* 2015; Mousavimehr *et al.* 2021]. The majority of cross-waves occur in supercritical flows, as well as in channels with asymmetrical alignments and non-prismatic cross-sections. Cross-waves occur as an effect of the deflected wall, causing a non-uniform flow [Akmal 2014]. The deflection in side-channels can block flows, and the deflected inner wall bends away from the flow. The disturbance lines generated by the outer and inner walls are reflected alternately between the two walls. The reflection affects the water level which varies from maximum to minimum. These disturbance lines interfere with each other resulting in cross-waves [Chow 1992]. Previous studies show that a straight construction is better than a deflected structure based on the length of the disturbance line and the maximum water level at an exact location [Ippen, Dawson 1951]. A supercritical flow passing through a straight construction generates symmetrical shock waves that propagate along the stream until they reach the opposite wall. However, in a deflected construction, cross-waves interfere with each other to form new disturbance patterns moving towards the downstream channel. Cross-waves always occur in almost all cases of side-channel spillways [Legono *et al.* 2019]. Cross-waves are highly probable to occur if space downstream the side-channel spillway is narrower and may cause water levels around the cross-waves to rise [Ali, Yousif 2019].

Cross-waves can generate pulsating flows, i.e. a hydraulic phenomenon in a chute channel. The phenomenon causes irregular flows leading to a hydrodynamic force that endangers the stability of the construction. Furthermore, the flow velocity at the foot of the chute channel becomes irregular, thereby reducing the dissipation effectiveness [Elnikhely 2018]. Instabilities may occur in a long chute channel, which leads to slugs/pulsating flows. If the length of the channel is more than 30 m, it must be controlled by checking the Vedernikov number and the Montouri number [USBR 1978]. The relationship between the two values is analysed as regards the pulsating flow region. To eliminate the pulsating flow, structure design modifications need to be done if the results of the analysis indicate the pulsating flow area.

From the approach channel to the escape channel, flow conditions in spillway systems cannot be adequately predicted through analytical calculations and mathematical models at the planning stage. Therefore, it is necessary to test several features of their construction to determine optimal conditions. In this study, a comparative investigation of different side-channel spillway models was carried out to obtain a high confidence level. This study aimed to analyse the flow rate effect on the formation of cross-waves through a series of tests in the laboratory. First, it examined the water depth by measuring the water surface, an elevation profile, to find the position of the starting point of air entry and to describe the occurrence of cross waves in the flume for original series. Next, the model for original series was modified into subsequent series to obtain optimal hydraulic conditions. The experiment was recorded laterally for all series to evaluate the effect on the free water surface.

MATERIALS AND METHODS

The flume experiments were carried out using a physical model of a side-channel spillway built at the River and Coastal Experiment Laboratory of Syiah Kuala University, Indonesia (Fig. 1). The model used a undistortion ratio on the scale of 1:60. The length of the prototype transition channel was 300 m and the prototype chute channel was 150 m. We obtained geometric (length, width, area, and depth) and kinematic (time, velocity, flow) similarities, which were then applied to the model. Froude numbers are equally valued between the model and the prototype.

Water is pumped from the underground reservoir into an open channel upstream of the spillway and returned in a closed circuit. The flow between the reservoir and the open channel is controlled by a rectangular rechbox, acting as a flow meter that distributes and aligns the flow. The spillway model is constructed according to the engineering design [Dinas Sumber Daya Air 2020].

Measurements were carried out on the water level, velocity, and cross-wave distance, height, and movements for each channel grid planned. Height and distance of cross-wave at each grid was plotted using Surfer. Surfer is a software used for contouring maps and three-dimensional modeling based on grids. This software plots irregular *XYZ* tabular data into a sheet of irregular rectangular grids. Measurement of flow velocity and profile was done to assess the occurrence of a pulsating flow. Data were collected at three parts, i.e. right, middle, and left. The water level and the average velocity were used to depict the height and slope of the water surface to obtain an energy loss value (*Hf*) at each discharge. Furthermore, the gradient energy angle (θ) was further obtained by the analysis of a pulsating flow. A flow simulation of inflow and outflow discharge was employed to ensure the measurement accuracy of the model. The simulation served as a model calibration to create an accurate discharge. Piezometric

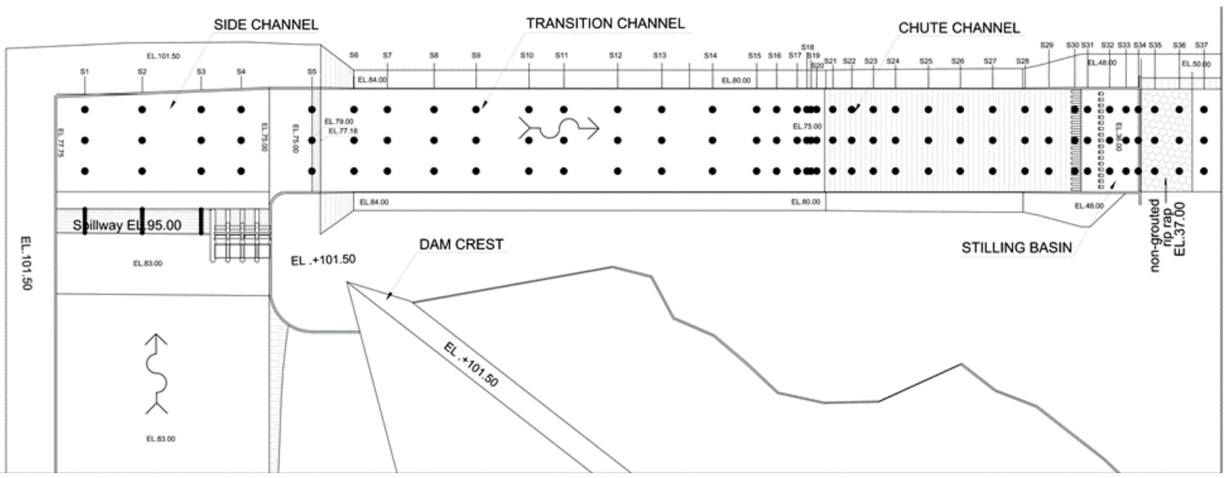


Fig. 1. Experimental installation and measurement points; source: own elaboration

calibration was performed through a simulation between measuring and checking calculation results. The accuracy of this measuring tool is necessary to create the same model conditions as the prototype. The water level was measured using a point gauge and the flow velocity was generated by comparing the water level at the foot of the piezometer tube (*ΔH*). The piezometer is based on the Bernoulli's principle. The Bernoulli's principle defines how the velocity of a fluid relates to its pressure. The water level and flow velocity on the grid threads are shown in Figure 1, and a long profile of the experimental installation is depicted in Figure 2.

There are several differences between the design of original and modified series. It includes a 4.2 m decrease of the side-channel ground elevation, construction of a 4.5-meter-wide sill on the side-channel at 31 m from the opening of the transition channel, and the sill is shifted from the opening of the transition channel to the approach channel, approximately 25 m from the end of the side-channel. Moreover, the wall at the beginning of the transition channel was 3 m high, and then the transition channel wall and the launcher channel wall were 1 m high.

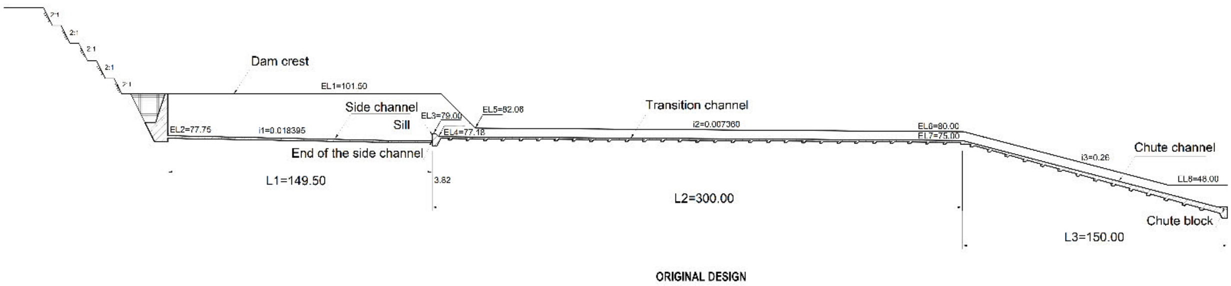


Fig. 2. Long profile of the experimental installation; source: own elaboration

The requirements to obtain optimal conditions for the transition channel and chute are sufficient freeboard for the flow to pass, an evenly distributed flow velocity along the channel, and the absence of cross-waves and pulsating flow. To find out the onset of pulsating flow symptoms using the Vedernikov number and the Montouri number.

The Vedernikov number ($\underline{V}$)

$$\underline{V} = \frac{2bv}{3P\sqrt{gd\cos\theta}} \tag{1}$$

The Montouri number ($\underline{M}$)

$$\underline{M}^2 = \frac{V^2}{gIL\cos\theta} \tag{2}$$

where: b = width of bottom channel (m), v = average of velocity (m·s^{-1}); P = hydraulic perimeter (m); g = gravity acceleration (9.81 m·s^{-2}), d = average of water depth (m), θ = angle of energy gradient, I = average of energy gradient slope = tan θ, and L = flow length (m).

The calculated values using both equations are further explored on the pulsating flow criteria graph to find out the onset of the pulsating flow. The graph is shown in Figure 3.

Furthermore, if the calculation point is in the area or region where the pulsating flow occurs, the form factor d/P must be calculated. Results are re-plotted onto Figure 4, which specifically shows a pulsating flow based on the shape of transition and chute. Waves arise only when the points are located within the vibration area in both pictures. If the figure shows that a pulsating flow occurs, the design can be modified to reduce the likelihood of the resulting wave or the structure can be adjusted to accommodate the flow of the surge. Modification is made by changing the slope or width of the channel. If this is not possible, it is necessary to provide an energy reducer pool using an additional free board and a wave suppressor.

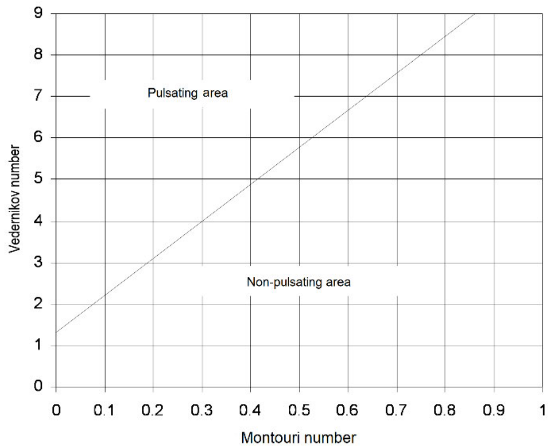


Fig. 3. Pulsating flow criteria graph; source: USBR [1987]

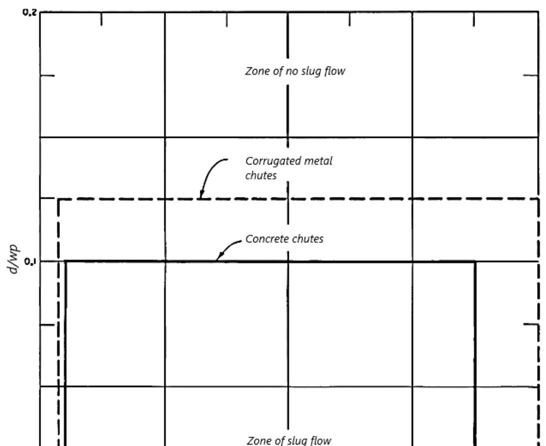


Fig. 4. Pulsating flow shape and criteria; d = average of water depth (m), θ = angle of energy gradient, wp = wet perimeter; source: USBR [1987]

RESULTS AND DISCUSSION

This section presents the hydraulic flow conditions of the spillway, which was further analysed. Flow patterns were evaluated for all flow rate conditions. Physical model testing was carried out on three variations of flow rates (with return periods), i.e., Q_2, Q_{100}, and Q_{1000} (notations valid for prototype) with flow rates of 5.72, 16.62, and 18.91 $m^3 \cdot s^{-1} \cdot m^{-1}$, respectively. Resulting flow rates are products of hydrological analysis at the study location [Water Resources Department of Aceh 2020]. Results of flow tests on the original series are shown in Table 1 and Figure 5.

Table 1. Distance and height of cross-wave (original series)

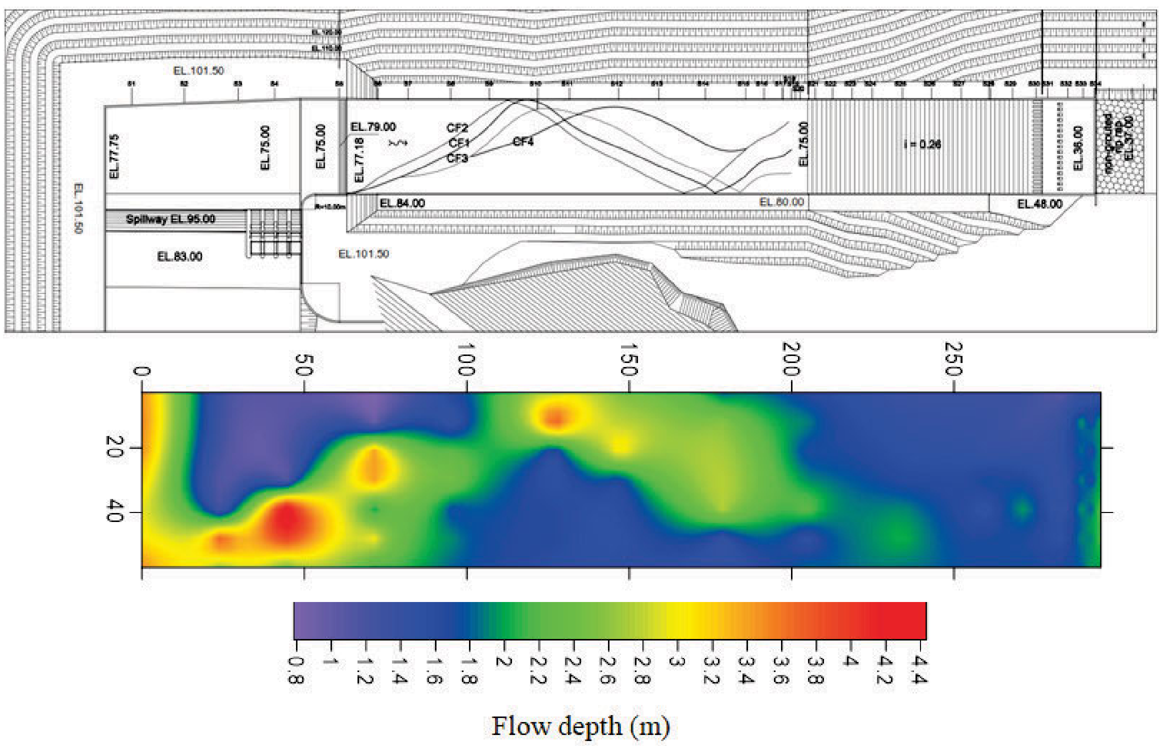
No.	Flow discharge ($m^3 \cdot s^{-1}$)	Distance of the first cross-wave from the transition channel (m)	Height of the first cross-wave (m)	Distance of the second cross-wave from the transition channel (m)	Height of the second cross-wave (m)
1	Q_2	116.0	3.3	226.6	2.2
2	Q_{100}	81.4	4.2	198.8	3.8
3	Q_{1000}	79.0	5.0	194.3	4.0

Source: own study.

Further information on flow patterns at the left and right walls of the transition channel needs to be examined from Figure 5. In general, the flow conditions along the spillway system were highly turbulent, causing irregularity of the water surface in the form of cross-waves. This condition, which happened at all flow rates (from Q_2 to Q_{1000}), caused a thicker flow to form on the left side of the transition channel wall, approximately 96 m from the side-channel and transition channel. Based on the streamlines at a flow rate of Q_2 (see Fig. 5a), the left and right walls have high performance, which corresponds to the cross-waves formed. However, the increase in flow rates from Q_{100} to Q_{1000} led to the left wall performance that does not correspond to the abundance of cross-waves. The geometric design created secondary flows and movements that resulted in cross-waves propagating downstream to the right wall (Fig. 5). The left wall of the transition channel formed initial waves that raised the water surface adjacent to the wall where the wave hit and continued with the next waves. An impact cross-wave was formed on the left wall, approximately 138 m after the previous wave. Based on the experiment, high flow rates led to cross-waves formed closer to the sills of the transition channel. This result was consistent with the study by Ali and Yousif [2019].

Cross-waves are formed due to water flowing from the side-channel to the transition channel. A strong interaction occurs between flows and vortexes that circulate in the side-channel and continue to the transition channel due to an introduction of a large amount of air into the main current [Ali, Yousif 2019]. The mix of water flows from the side-channel is irregular. An incomplete energy dissipation process at the side-channel results in hydrodynamic forces as flow impacts and vibration forces happening in the channel. Based on stability considerations, the channel is recommended to be built on a solid rock foundation. It is also recommended that the width of the side-channel reduced with a narrower base to generate a more significant energy dissipation effect [Sosrodarsono, Takeda (eds.) 1981]. The water surface profile for the design discharge determines the height of the transition wall, considering the high cross-waves formed. The required transition wall height should refer to the classic spillway design in "Small dams design" [USBR 1987] and "Spillways hydraulic design" [USACE 1990]. Meanwhile, it is more difficult

a)



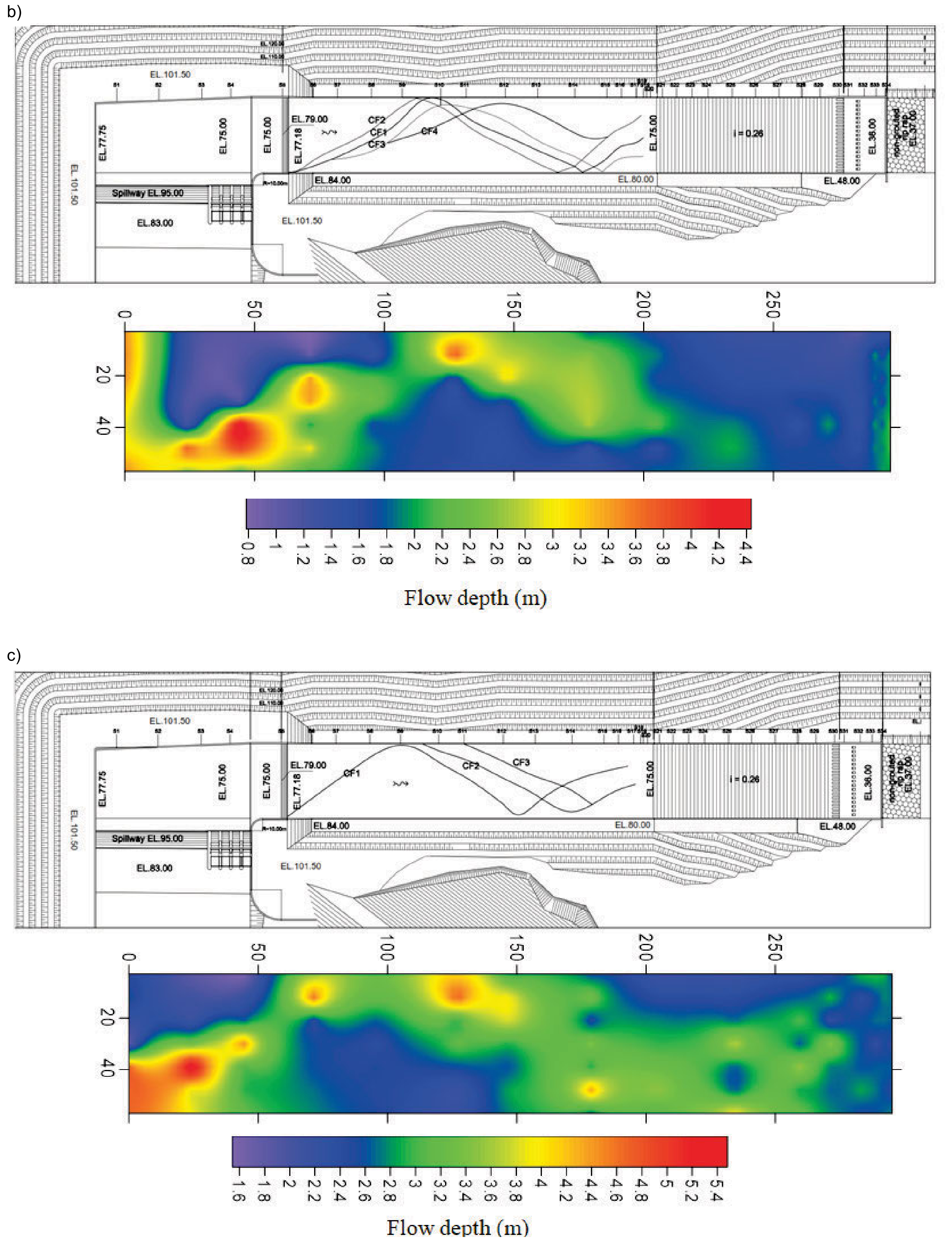


Fig. 5. Cross-waves at three different flow rates (original series): a) Q_2 = 5.72 $m^3 \cdot s^{-1} \cdot m^{-1}$, b) Q_{100} = 16.62 $m^3 \cdot s^{-1} \cdot m^{-1}$, c) Q_{1000} = 18.91 $m^3 \cdot s^{-1} \cdot m^{-1}$; source: own study

to determine the transition height of contracting side-channel spillways. To predict the performance of spillway systems and determine the wall height, physical model testing is required due to sharp contraction [PCA 2002].

Cross-waves are unfavorable for the downstream flow because they can cause non-uniform flow and incomplete energy dissipation in the stilling basin, which eventually erodes the river downstream. Therefore, spillway systems need to be modified. Modification of the spillway system is based on the magnitude of height and distance of a cross-wave on the transition channel wall. The modifications helped to eliminate cross-wave and non-uniform flows through decreasing the side-channel ground elevation, adding a sill on the side-channel, and moving the sill from the opening of the transition channel to the approach channel [Legono *et al.* 2019]. The modifications were done to obtain optimal hydraulic conditions and eliminating cross-wave at transition channel and chute. Multiple laboratory experiments need to be carried out to determine the sills' optimal position and geometric shape [Prasetyorini *et al.* 2020]. The final modifications applied to the last series were a 4.2 m decrease of the side-channel ground elevation, construction of a 4.5-meter-wide sill on the side-channel approximately 31 m from the opening of the transition channel, and moving the sill from the opening of the transition channel to the approach channel approximately 25 m from the end of the side-channel. The additional wall at the beginning of the transition channel was 3 m high and the transition channel wall and the launcher channel wall were 1 m high. Possibilities for cross-wave occurrence were analysed again. Figure 6 shows the flow results.

Figure 6 shows that cross-waves and non-uniform flows on the left and right sides of the flume have been eliminated. The proposed physical model modifications demonstrated a preferable performance by having the flow pass freely through the left and right walls of the transition channel. The flow profile is symmetrical on the centerline of the transition and chute channel, which proved that the modified series provided an efficient downstream flow due to flow alignment. These findings help to understand the cross-wave behavior formed earlier in locations closer to the wall rather than in the middle of the flume. The flow field shows a periodic pattern formed by cross-waves on the left and right walls of the transition channel [Lopes *et al.* 2015].

The pulsating flow analyses in discharge variation for the original series and modified series were shown in Figure 7.

The graphs for the original and modified series in Figure 7 show that no Vedernikov and Montouri numbers are recorded in

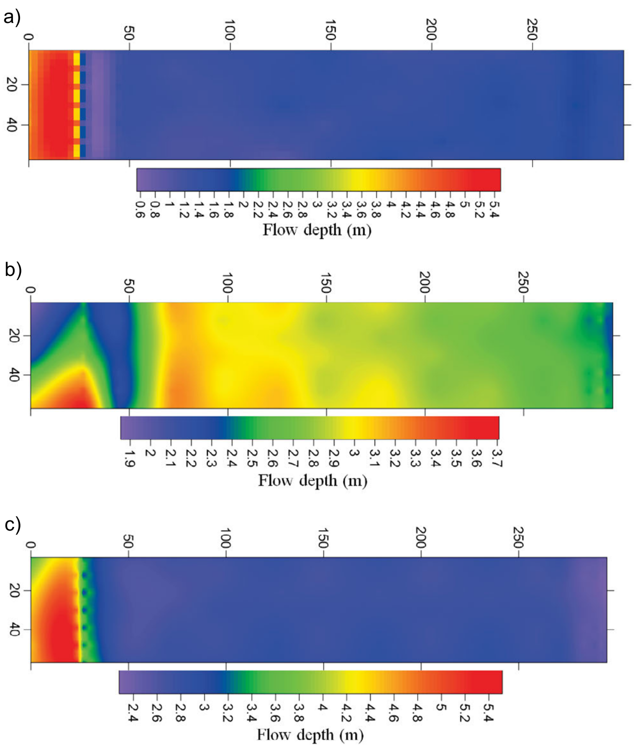


Fig. 6. Cross-waves after three physical model modifications (modified series): a) Q_2 = 5.72 $m^3 \cdot s^{-1} \cdot m^{-1}$, b) Q_{100} = 16.62 $m^3 \cdot s^{-1} \cdot m^{-1}$, c) Q_{1000} = 18.91 $m^3 \cdot s^{-1} \cdot m^{-1}$; source: own study

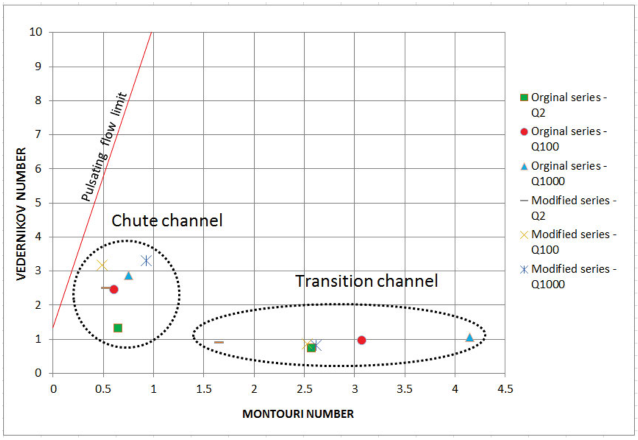


Fig. 7. Pulsating flow analysis in discharge variation for original and modified series; Q_2, Q_{100}, Q_{100} for original series as in Fig. 5 and modified series as in Fig. 6; source: own study

the area where the pulsating flow occurs. It means that both series are free from pulsating flow. Specifically, the modified series was closer to the pulsating flow area (launch and transition) because the transition channel and launcher in the modified series were longer than in the original series. A long, steep-angled glide channel endangers the stability of a pulsating flow [Cassidy 1990; Di Cristo *et al.* 2010]. Wibowo [2016] mentioned that if the channel is more than 30 m, the Vedernikov number ($\underline{V}$) and Montouri number must be controlled. This study was strengthened by Di Cristo *et al.* [2010], arguing that the Vedernikov's theory systematically overpredicts the pulsating flow for steep channel slopes. The model modification was done to meet the Montouri criteria with a minimum channel length. But based on the results of this study there was no pulsating flow, so it was not necessary to make modifications by changing the slope or width of the channel to reduce the pulsating flow. Modification was only needed to reduce the cross-wave effect.

The energy gradient is also a parameter to determine the pulsating flow based on the flow rate. The modified series provides a lower energy gradient than the original ones. The cross-waves formed in the transition channel of the original series affected the launcher channel. Changes in the flow profile due to the changes in the flow height occurred irregularly, causing the flow speed to be irregular. This finding was in line with research results by Julien and Hartley [2010], who reported that cross-waves and pulsating flow applied to turbulent flow. This analysis also showed that the distance was equal to the ratio of flow depth and slope for the supercritical flow. Changes in flow height and velocity made the flow discharge points on the graph look stacked and close. These points should spread out and increase with the flow rate. The cross-waves are likely to make the flow vibrate so that under these conditions, the flow is not favorable downstream. Cross-waves can lead to irregular flow and imperfect energy dissipation in stilling ponds. This may eventually lead to scouring in the river downstream. Therefore, cross-waves and pulsating flow must be omitted by modifying the spillway system. Various experiments need to be carried out in the laboratory for the positioning and geometrical shape of the sill at the beginning of the transition channel to obtain the best results [Prasetyorini *et al.* 2020].

CONCLUSIONS

This study demonstrates the hydraulic behavior, effect of cross-waves, and pulsating flow in the side-channel spillway. Cross-waves occurred in all flow rate variations in the original series of the experiment. Although the starting point of Q_2 discharge was detected by slight turbulence on the water surface at the left wall of the side-channel, about 1.62 m from its upstream end, overtopping on the wall did not occur. Downstream cross-waves occurred on the right wall crosswise. The flow depth of the right wall was lower than that of the left wall. The cross-wave flow height at Q_{100} to Q_{1000} increased substantially, resulting in overtoppings on the left and right walls. The flow depth at the right wall remained lower than that at the left wall. Several modifications were made to the spillway system, i.e., a 4.2 m decrease of the side-channel ground elevation, construction of a 4.5-meter-wide sill in the side-channel approximately 31 m from the opening of the transition channel, and moving the sill from the opening of the transition channel to the approach channel approximately 25 m from the end of the side-channel. The wall added at the beginning of the transition channel was 3 m high and the transition channel wall and the launcher channel wall were 1 m high. These modifications provided optimal hydraulic conditions, i.e. eliminated cross-waves and non-uniform flows along the transition channel. The Vedernikov and Montouri numbers showed that both original and modified series did not enter the area where the pulsating flow occurred. This indicated that both series were safe from a pulsating flow.

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