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An empirical model of rainfall intensity as a function of rainfall duration and probability of occurrence

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Abstract: Rainfall is one of the main components of the hydrologic cycle; thus, the availability of accurate rainfall data is fundamental for designing and operating water resources systems and infrastructure. This study aims to develop an empirical model of rainfall intensity ($I_{t,p}$) as a function of its probability (p) and duration (t). In 1999–2020, data on the hourly duration of rainfall were collected from automatic rainfall recorder (ARR) gauges. The empirical model has been developed using a statistical approach based on duration (t) and probability (p), and subsequently they have been validated with those obtained from ARR data. The resulting model demonstrates good performance compared with other empirical formulas (Sherman and Ishiguro) as indicated by the percent bias (*PBIAS*) values (2.35–3.17), ratio of the *RMSE* (root mean square error) between simulated and observed values to the standard deviation of the observations (*RSR*, 0.028–0.031), Nash–Sutcliffe efficiency (*NSE*, 0.905–0.996), and index of agreement (d, 0.96–0.98) which classified in the rating of "very good" in model performance. The reliability of the estimated intensity based on the empirical model shows a tendency to decrease as duration (t) increases, and a good accuracy mainly for the rainfall intensity for shorter periods (1-, 2-, and 3-hours), whereas low accuracy for long rainfall periods. The study found that the empirical model exhibits a reliable estimate for rainfall intensity with small recurrence intervals (*Tr*) 2-, 5-, 10-, and a 20-year interval and for a shorter duration (t). Validation results confirm that the rainfall intensity model shows good performance; thus, it could be used as a reliable instrument to estimate rainfall intensity in the study area.

Keywords: empirical model, probability of occurrence, rainfall duration, rainfall intensity, recurrence interval

INTRODUCTION

A better understanding of rainfall characteristics becomes a primary requirement for enhancing accuracy in designing and operating water-resources systems [Harisuseno *et al.* 2020]. High rainfall intensity is believed to have a fundamental disastrous impact on management in a basin [Guo *et al.* 2016; Hong *et al.* 2018]. Due to the unavailability and consistency of rainfall data, the designing of water resources structures is difficult to handle. The lack of rainfall data leads to an inaccurate design of hydraulic structures that are prone to design failure. Thereby, engineers must be able to quantify rainfall since it is an important input for predicting design discharge while planning a system of water collection, conveyance, and storage [Harisuseno *et al.* 2019].

Rainfall intensity-duration-frequency (*IDF*) is one of the primary inputs for designing a drainage system and other water resources infrastructure [Jun *et al.* 2017]. Previous studies have been carried out to determine the relationship between intensity, duration, and frequency of rainfall. Rasel and Hossain [2015] produced empirical models using the statistical Gumbel distribution to predict rainfall intensity for various recurrence intervals and durations in Bangladesh. Zope *et al.* [2016] developed an *IDF* curve for Mumbai, India, using annual maximum daily rainfall based on empirical formulae. Blanchet *et al.* [2016] introduced the *IDF* based on a regional generalized extreme value (*GEV*) scale-invariant framework, whereas Ghanmi *et al.* [2016] produced a regional *IDF* for Tunisia based on the Gumbel distribution. Soltani *et al.* [2017] extracted *IDF* curves using a simple scaling model to determine the regionalization of the

IDF in southwest Iran. Since most rainfall data are provided based on daily periods, it is necessary to develop an *IDF*, particularly in regions that experience a lack of shorter rainfall events. Besides, hydrological information, such as the *IDF* relationship, is not yet readily accessible to water resource engineers, in particular in developing countries [Wagesho, Claire 2016]. Therefore, adequate information on rainfall intensity for certain recurrence intervals and durations must be available, especially in ungauged areas, to develop appropriate and efficient water resource planning. Further, there is a need to develop a rainfall intensity model compatible with rainfall characteristics in the upstream area of the basin. This study aims to develop a model of rainfall intensity as a function of probability (p) and duration (t) using a statistical approach. The resulting model could be used to predict rainfall intensity for a given duration (t) and the probability of its occurrence (p).

MATERIALS AND METHODS

STUDY AREA

The study area is located in the upstream region of the Lesti River basin, Malang Regency, East Java Province, Indonesia. Geographically, the Lesti River basin is positioned between latitude 8°02'–8°12' S and longitude 112°42'–112°56' E. The upstream area of the Lesti River basin occupies 383.4 km^2 with a radial flow of basin shape and average land slope ranging between 8 and 45%. The land use in the basin includes primarily plantations (42%), followed by paddy fields (17%), forests (14%), residential (10%), and bush areas (17%). Most of the basin area is mountainous with an altitude of 235–4,676 m above sea level. Meteorologically, the basin area has tropical climate with two seasons annually, where the wet season from October to March and the dry season from April to September. The monthly mean temperature is 20.5–26.2°C, while mean annual rainfall 2820 mm. Figure 1 depicts the study area and the automatic rainfall recorder (ARR) used in the study.

METHODOLOGY AND DATA USED

In 1999–2020, the rainfall data with hourly duration were collected by automatic rainfall recorder (ARR) gauges in Dampit and Poncokusumo. In this study, duration means the unit length of time to collect rainfall readings from the ARR gauge. The minimum time interval is 1-hour rainfall, since the ARR gauge is designed to collect the rainfall with a 1-hour minimum duration. Hence, it is possible to obtain rainfall data for each duration from the ARR gauge by reading the graphic paper for 1, 2, 3, 4, and 5 h in the ARR instrument. Thus, 1-hour duration means the amount of rainfall collected from the ARR in 1 h based interval, namely 2, 3, 4, and 5 h. The rainfall events with minimum 0.5 mm for each duration were used as the lower boundary of the rainfall depth, considering that the rainfall of 0.5 mm is the minimum value that often occurs at both ARR gauges (high probability of occurrence). Thus, the present study only selects rainfall data with a minimum value of 0.5 mm (≥0.5 mm) as the selected data to build the empirical model. Since the rainfall intensity model is developed as a function of probability (p) and duration (t), the present study used various durations of rainfall (1, 2, 3, 4, and 5 h). To ensure good data quality, statistical testing procedures,

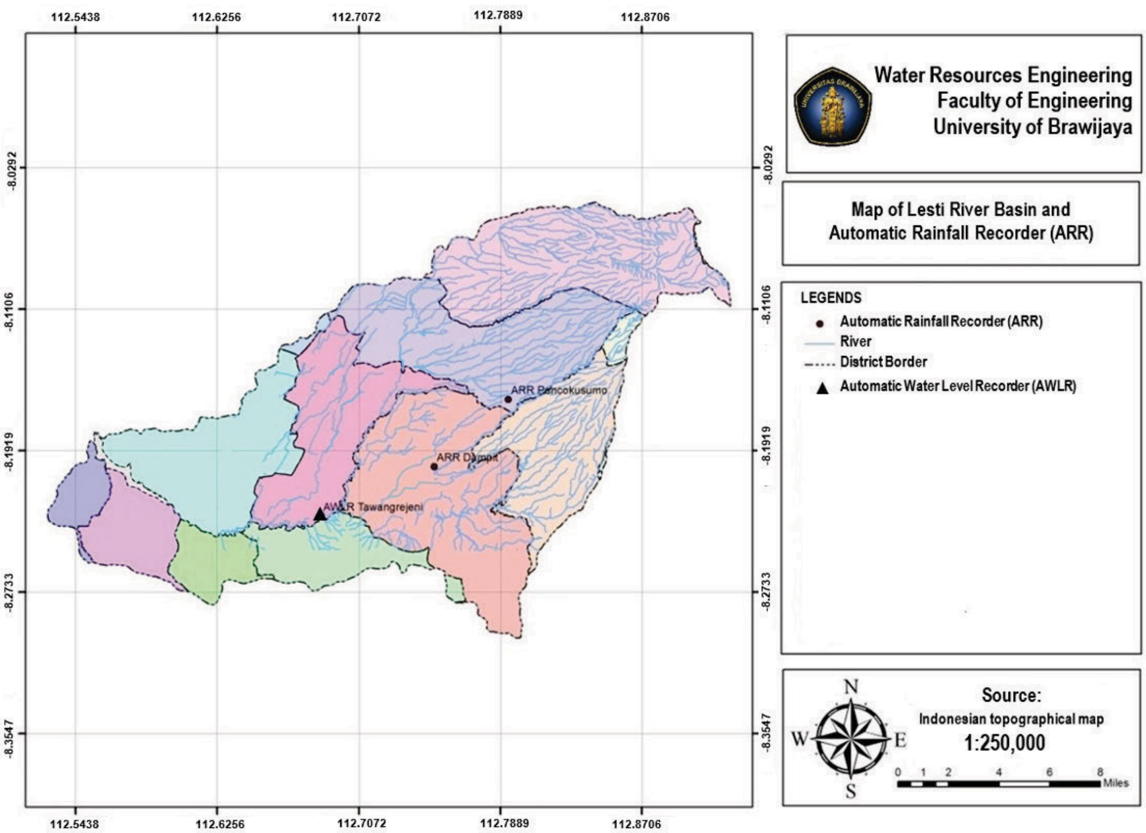


Fig. 1. Study area with rain gauge; source: own elaboration

e.g. consistency, normality, and homogeneity, were performed to examine rainfall [AlSaleem 2018]. For modelling needs, the present study used the mean areal rainfall calculated from both ARR gauges using the arithmetic method as the primary input to modelling.

The mean areal rainfall was then presented in a group according to its duration [da Silva *et al.* 2019]. The maximum duration of 5 h was chosen considering that most flood events were commonly caused by rainfall with a 5-hour duration in the study area [Adi 2013]. The empirical model of rainfall intensity was developed by considering the variation of duration (t) and probability of occurrence (p). Figure 2 presents the modelling of rainfall intensity employed in the present study.

The observed rainfall intensity for each duration (I) was computed by dividing the rainfall by its duration [Limantara *et al.* 2018; Soekarno, Rohmat 2006]. The probability of occurrence (p) of observed rainfall intensity for each duration was calculated using the Weibull equation [Dorneles *et al.* 2019]:

$$p = \frac{m}{n+1}100 \tag{1}$$

where: m = rank-ordered data, n = total number of data.

Since the model was developed using a statistical approach, the observed rainfall intensity must be tested for its normal characteristics using the Shapiro–Wilk test [Seyam, Othman 2014]. The empirical rainfall intensity model was validated by comparing the intensity for varying probability of occurrence (p) and duration (t) taken from the ARR data. Further, the validated model was examined for its reliability using other intensity formulas and estimating rainfall intensity of varying recurrence intervals (Tr) and duration (t).

SHERMAN AND ISHIGURO FORMULA

The reliability of the empirical model was investigated against the other formula, namely Sherman and Ishiguro, considering it is commonly used in the upstream area of the basin [Gutierrez-Lopez *et al.* 2019]. The Sherman formula can be derived from Equation (2), while Equation (3) denotes the Ishiguro formula.

$$I = \frac{a}{t^n} \tag{2a}$$

$$\log a = \frac{\sum \log I \cdot \sum (\log t)^2 - \sum(\log t - \log I)\log \sum t}{N\sum (\log t)^2 - \sum(\log t)(\log t)} \tag{2b}$$

$$n = \frac{\sum \log I \log \sum t - N\sum(\log t - \log I)}{N\sum (\log t)^2 - \sum(\log t)(\log t)} \tag{2c}$$

$$I = \frac{a}{\sqrt{t} + b} \tag{3a}$$

$$a = \frac{\sum(I\sqrt{t})\sum I^2 - \sum(I^2\sqrt{t})\sum I}{N\sum I^2 - \left(\sum I\right)^2} \tag{3b}$$

$$b = \frac{\sum(I\sqrt{t})\sum I - N\sum(I^2\sqrt{t})}{N\sum I^2 - \left(\sum I\right)^2} \tag{3c}$$

where: I = rainfall intensity (mm·h^{-1}), t = rainfall duration (min), a, b, n = constant parameters related to the meteorological conditions and rainfall characteristics (depth and duration).

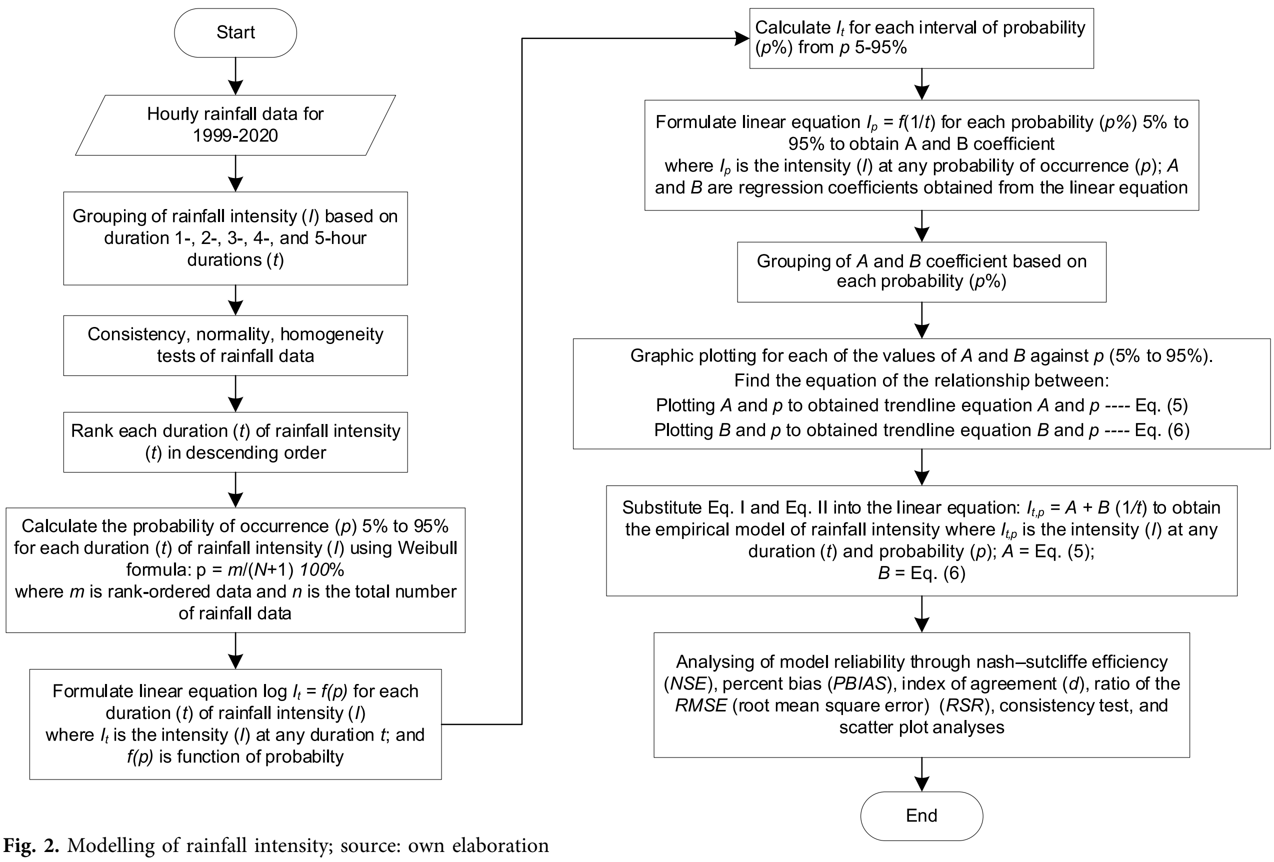


Fig. 2. Modelling of rainfall intensity; source: own elaboration

MODEL PERFORMANCE EVALUATION

The accuracy of the estimated intensity obtained from the empirical model, Sherman, and Ishiguro formula was examined by comparing it with the observed intensity yielded from log Pearson type III [Bhat *et al.* 2019]. The four statistical measures consist of percent bias (*PBIAS*), index of agreement (*d*), ratio of the *RMSE* (root mean square error) between simulated and observed values to the standard deviation of observations (*RSR*), and Nash–Sutcliffe efficiency (*NSE*). These were employed to assess the quality of the relationship [Moriasi *et al.* 2007; Wałęga 2016]. Table 1 presents performance ratings for each statistical measure used in this study.

Similar results were achieved by Tuama Al-Awadi [2016] who demonstrated that the intensity decreases with increasing rainfall duration while moving up for a high recurrence interval and shorter duration.

THE EMPIRICAL MODEL OF INTENSITY

The rainfall intensity (*I*) for the various probabilities of occurrence (*p*) at 1-, 2-, 3-, 4-, and 5-hour duration was determined based on the linear trend equation formulated from graphic plotting between log *I* and probability of occurrence (*p*) for each rainfall duration (*t*) as shown in Figure 3. Based on the linear equation presented in Figure 3, the intensity for various *p*

Table 1. Performance ratings for the statistics measures

Performance rating	*NSE*	*RSR*	*PBIAS* (%)	Index of agreement (*d*)
Very good	$0.75 < NSE \le 1.00$	$0.00 < RSR \le 0.50$	$PBIAS < \pm 10$	bounded by 0 (no agreement) and 1 (perfect fit)
Good	$0.65 < NSE \le 0.75$	$0.50 < RSR \le 0.60$	$\pm 10 \le PBIAS < \pm 15$	
Satisfactory	$0.50 < NSE \le 0.65$	$0.60 < RSR \le 0.70$	$\pm 15 \le PBIAS < \pm 25$	
Unsatisfactory	$NSE \le 0.50$	$RSR > 0.70$	$PBIAS \ge \pm 25$	

Source: Moriasi *et al.* [2007]. Used with permission.

The reliability of the empirical model in estimating rainfall intensity was tested through comparative analysis and consistency test [Harisuseno, Cahya 2020] between the estimated rainfall intensity obtained from the empirical model and the observed one.

The quality of comparison of both rainfall intensities were subsequently assessed using the same statistical measures that were previously applied to examine the degree of suitability between both rainfall intensities.

RESULTS AND DISCUSSION

RAINFALL PROBABILITY OF OCCURRENCE

The mean areal rainfall obtained from both ARR gauges using the arithmetic method was used as the major input for modelling. There are 716 mean areal rainfall events for all durations (1–5 h) collected in 1999–2020. The rainfall data were grouped based on duration consisting of 213, 185, 132, 105, and 81 data for 1, 2, 3, 4, and 5 h rainfall duration. Then, the rainfall data for each duration were ranked in a descending order. Subsequently, the probability of occurrence (*p*) of each rainfall data for each duration was computed using the Weibull formula from Equation (1).

Results of data quality testing using Levene's test, Shapiro–Wilk test, and rescaled adjusted partial sums (*RAPS*) showed that rainfall data were consistent and fulfilled the assumption of homogeneity and normality as indicated by *p*-value > 0.05 at the 5% level of significance (*α*). Figure 2 presents the relationship between the probability of occurrence (*p*) and observed rainfall intensity for various durations (*t*) obtained from the Weibull formula. The figure demonstrates that the short rainfall duration is associated with high intensity that has a low probability of occurrence.

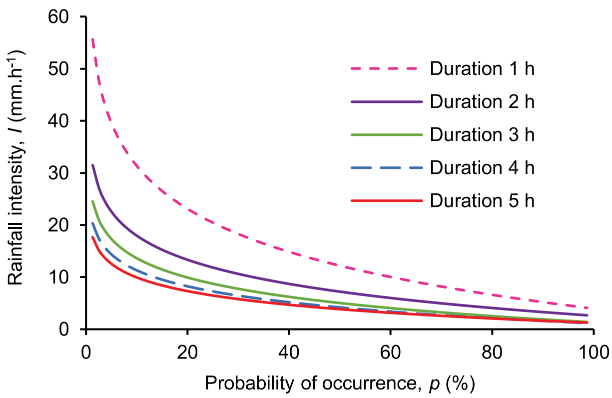


Fig. 2. Rainfall intensity (*I*) and probability of occurrence (*p*) for each duration (*t*) in 1999–2020; source: own study

(from 5 to 95% with intervals of 5%) and *t* could be derived. The following step determined the equation of the empirical model as a function of *p* and *t* based on values obtained from the linear equation shown in Figure 4. The varying rainfall intensities (*I*) were then plotted against all duration (1/*t*) for 1- until 5-hour duration for each probability of occurrence (*p*) in a range of 5 to 95%. Hence, we have nineteen linear equations that explain the relationship between rainfall intensity (*I*) and duration (1/*t*) for the probability value of 5 to 95%. Figure 5 exhibits an example of the relationship between intensity (*I*) and duration (1/*t*) for the probability of occurrence (*p*) 5 to 95%. As displayed in Figure 4, there was a linear relationship between intensity (*I*) and duration (1/*t*).

$$I = A \pm B(1/t) \qquad (4)$$

where: *A* and *B* = regression coefficients obtained from the nineteen of the linear equation.

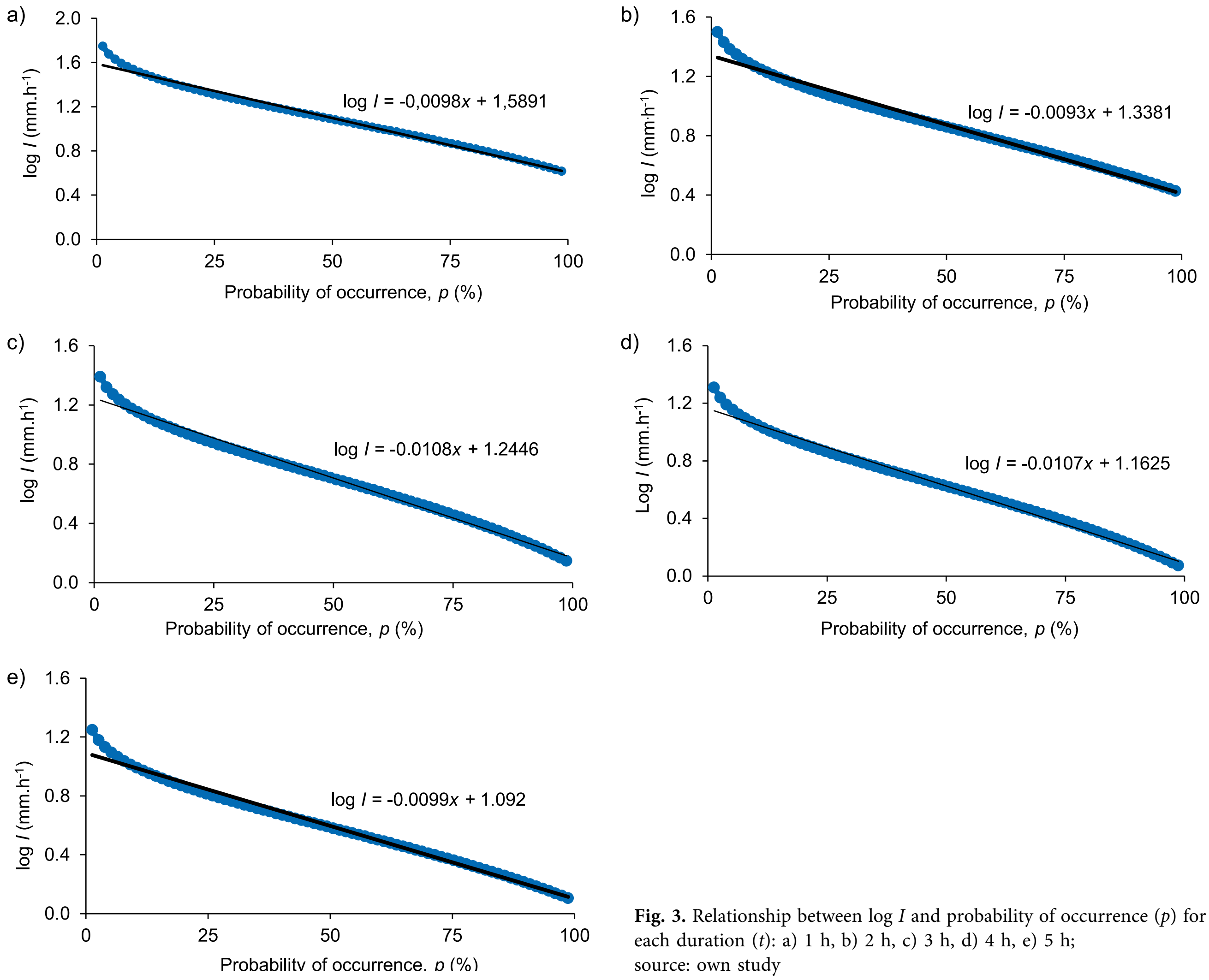
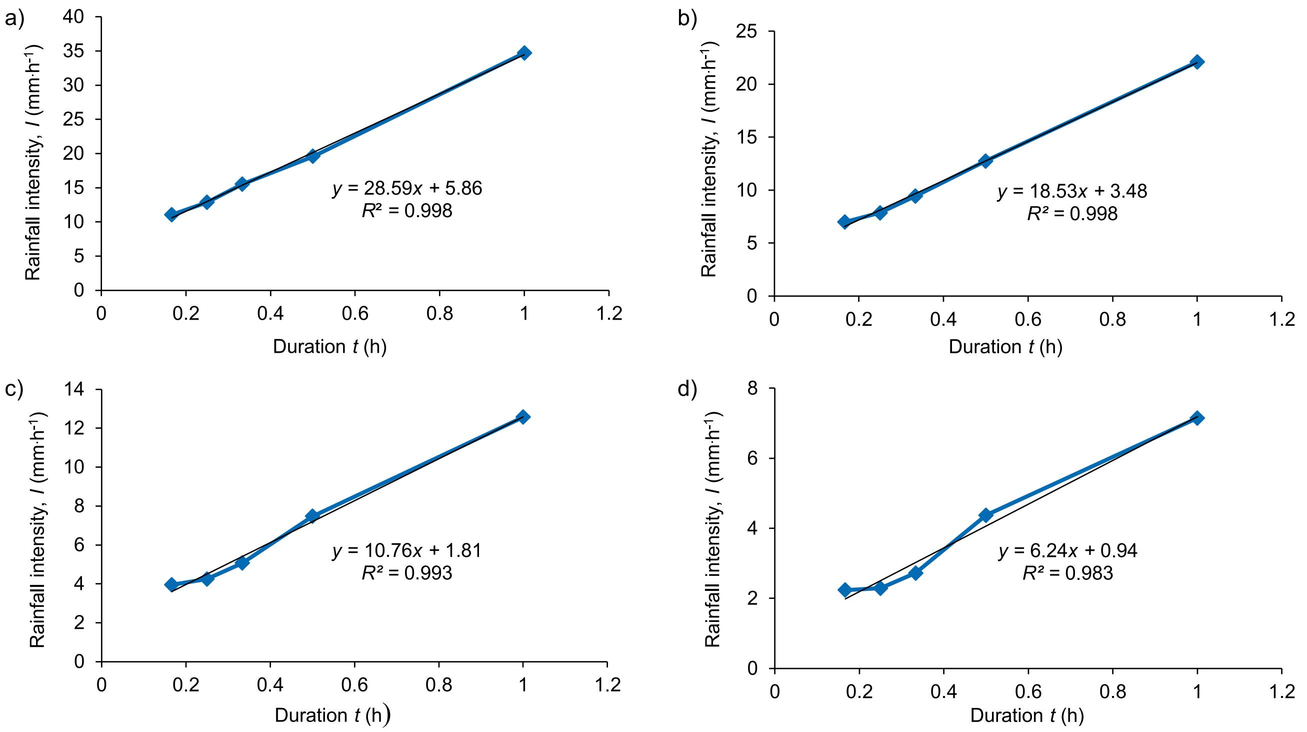


Fig. 3. Relationship between log *I* and probability of occurrence (*p*) for each duration (*t*): a) 1 h, b) 2 h, c) 3 h, d) 4 h, e) 5 h; source: own study



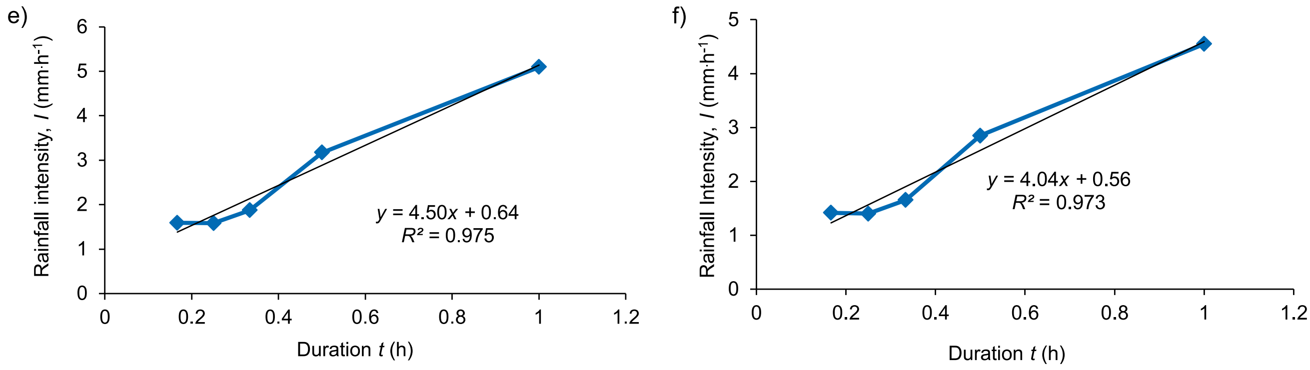


Fig. 4. Example of the relationship between intensity (I) and duration ($1/t$) for the different probability of occurrence (p): a) 5%, b) 25%, c) 50%, d) 75%, e) 90%, f) 95%; source: own study

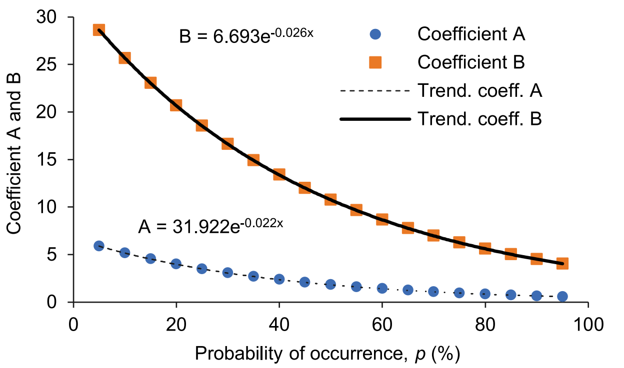


Fig. 5. Relationship between the probability of occurrence (p) and regression coefficients A and B from the nineteen linear equation; source: own study

Afterward, the graphic plotting between 19 linear regression coefficients for each value of A and B against p (5–95%) was made as demonstrated in Figure 5. Accordingly, the equation of coefficients A and B could be derived from Figure 5 as follows:

$$A = 31.922e^{-0.022p} \tag{5}$$

$$B = 6.693e^{-0.026p} \tag{6}$$

Thus, the empirical model of rainfall intensity equation could be obtained as follows:

$$I_{t.p} = 31.922e^{-0.022p} + 6.693e^{-0.026p}(1/t) \tag{7}$$

VALIDATION OF THE EMPIRICAL MODEL

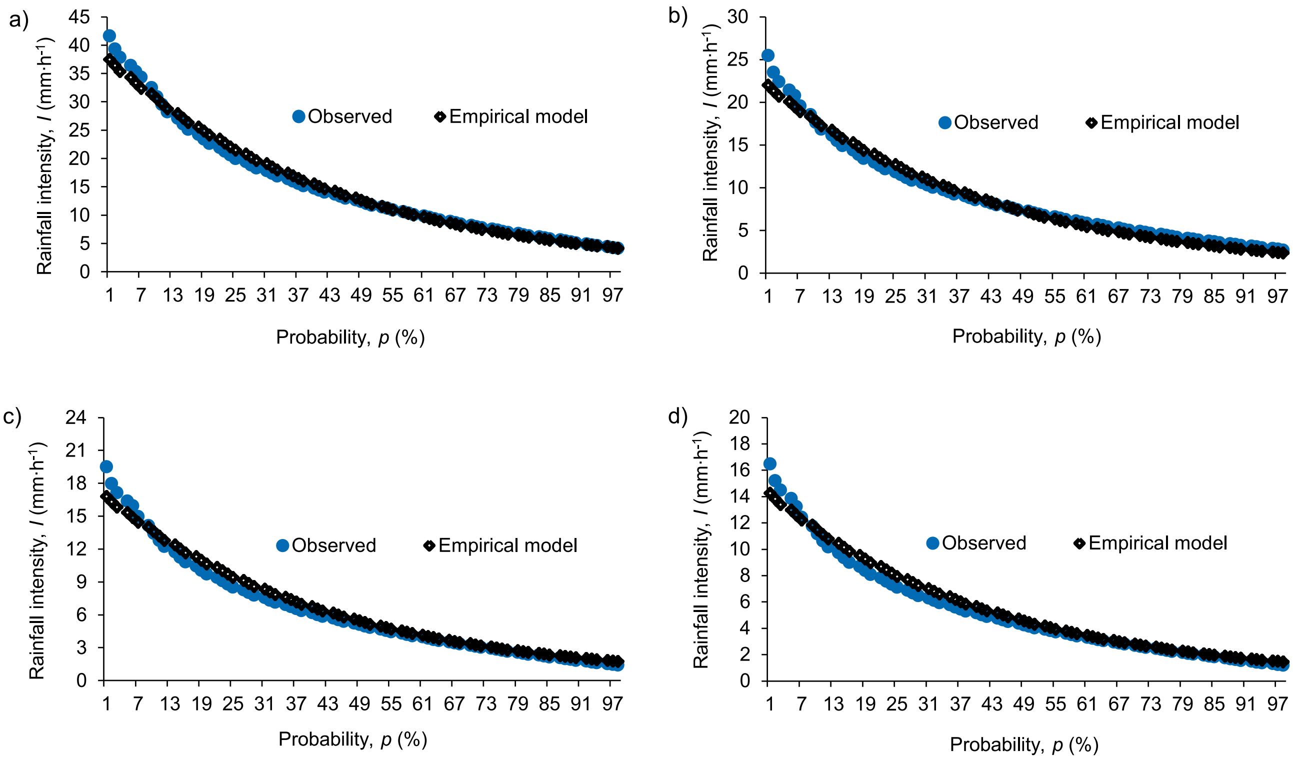
The validation of the empirical model was carried out to examine the relationship between the estimated intensity from the empirical model and the observed intensity for various probabilities of occurrence (p). The probability of occurrence (p) for a particular observed intensity was estimated by sorting in a descending order from high to low intensity for each group of t and subsequently the probability of occurrence (p) was computed using the Weibull equation [Dorneles *et al.* 2019]. As explained previously, the empirical model was formulated as a function of t and p; thus, the intensity in various t and p could be easily obtained by putting given t and p into the empirical model. The comparison between the estimated and observed intensity is presented in Figure 6.

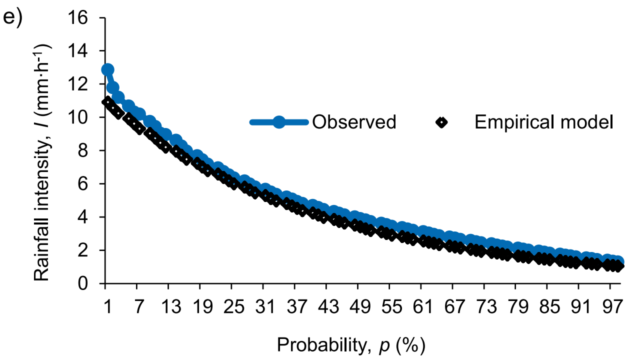
As depicted in Figure 6, the curve of rainfall intensity vs probability of occurrence (p) from both estimated and observed rainfalls exhibits a satisfactory relationship, as indicated by the congruence of both curves for all *Tr*. However, the relationship quality between the curve of estimated intensity from the empirical model and the observed one tends to decrease as t increases. The congruity between both curves shows an excellent fit for 1-, 2-, and 3-hour duration, while it seems to decrease for 4- and 5-hour duration. Typically, the high rainfall with a longer t has a small p. Thus, consequently, the series number of available rainfall data used to build the empirical model is not as good as the series of rainfall with a shorter t quantitatively and qualitatively. Commonly, the high rainfall with a short duration (t) in a rainy season occurs frequently due to its high probability of occurrence (p). Consequently, the feasible number of rainfall data series is available. Hence, it increases the accuracy of the empirical intensity model developed. Therefore, the empirical model results show good accuracy in estimating the intensity for shorter t (i.e. high p), and in contrast, low accuracy for long-duration events (i.e. small p).

Figure 7 presents the consistency test through a scatter plot of the cumulative intensity from the observed intensity as the x-axis and the empirical model as the y-axis. The consistency is achieved when the scatter points are situated around or near the straight line (45°) or 1:1 line. As exhibited in Figure 7, the intensity from the empirical model and the observed one show a consistent pattern as indicated by scatter points situated around the straight red line (45°), particularly for 1-, 2-, and 3-hour duration.

The intensity for 4- and 5-hour duration tends to show an inconsistent pattern, as indicated by scatter points that deviate from the straight red line.

This result confirms what has been mentioned previously that the empirical model is adequate to estimate intensity for a short duration (1, 2, and 3 h). Further analysis concerning the statistics measures as presented in Table 2 exhibits that the *NSE* values range from 0.965 to 0.989 while the index of agreement (d) values are 0.929 to 0.956. Since both are close to 1, it indicates the rating of "very good" in model performance as explained in Table 1. Furthermore, the *RSR* values are within 0.106 to 0.188, classified as "a very good" in the model performance rating. The *PBIAS* values range from –4.476 to –0.570, which likewise denotes a model





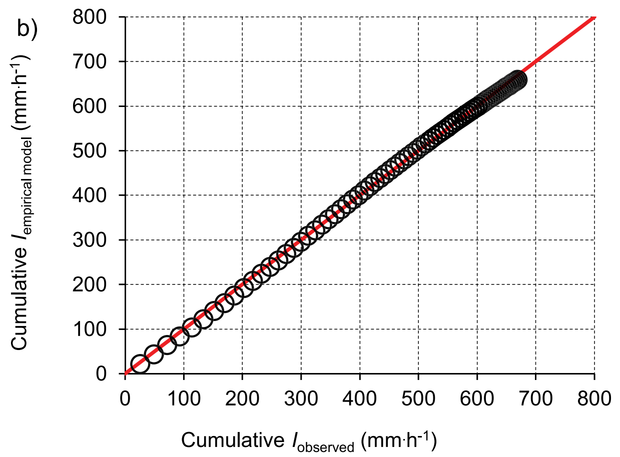
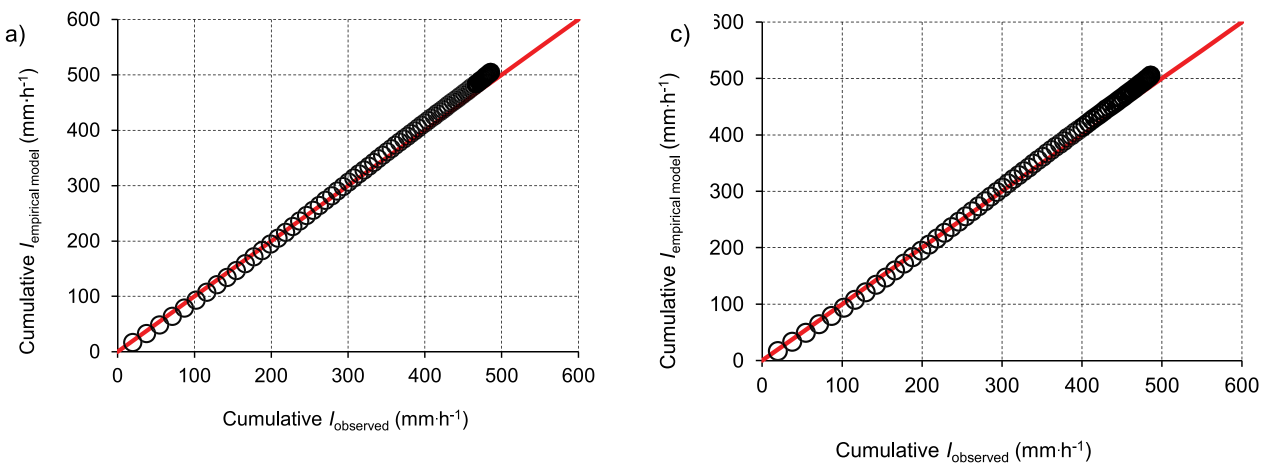


Fig. 6. The comparison between the estimated and observed intensity for the varying probability of occurrence (p) and duration (t): a) duration 1 h, b) duration 2 h, c) duration 3 h, d) duration 4 h, e) duration 5 h; source: own study



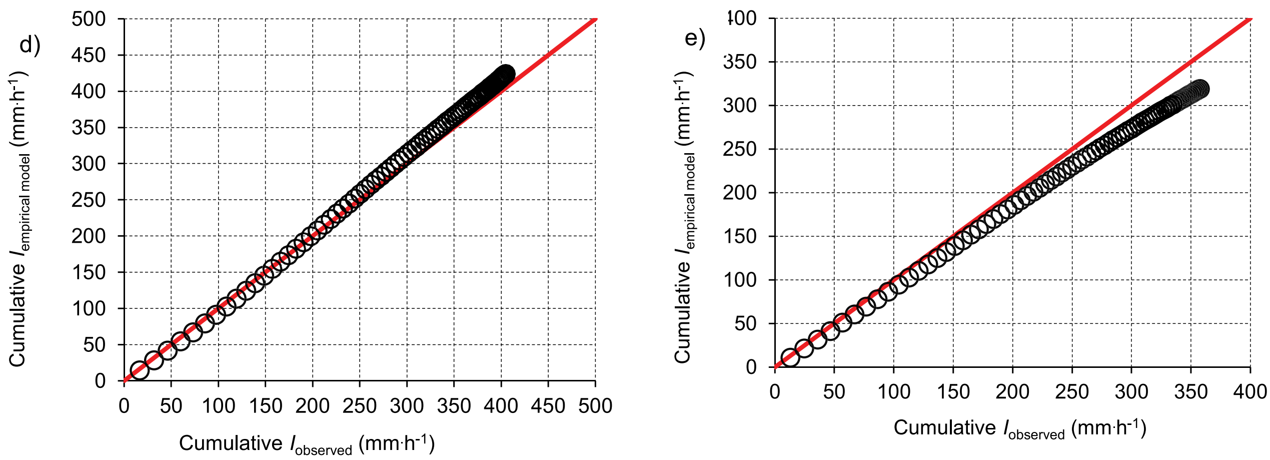


Fig. 7. The consistency test between the estimated from the empirical model and observed intensity for the various duration (*t*) and probability of occurrence (*p*): a) duration: 1 h, b) duration 2 h, c) duration 3 h, d) duration 4 h, e) duration 5 h; source: own study

Table 2. The statistics of validation of the empirical model

Duration *t* (h)	*NSE*	*PBIAS* (%)	Index of agreement (*d*)	*RSR*
1	0.989	–0.570	0.956	0.106
2	0.982	1.486	0.940	0.132
3	0.980	–4.048	0.932	0.143
4	0.977	–4.746	0.929	0.151
5	0.965	10.626	0.897	0.188

Explanations as in Tab. 1.
Source: own study.

performance rating of "very good" for all durations, except for the 5-hour duration (*PBIAS* of 10.626) that is classified as "good" in the model performance rating. Thus, the model validation results indicate that the empirical model shows good performance, which accordingly could be used as a capable and reliable instrument to estimate the rainfall intensity with good accuracy in the study area.

THE SHERMAN AND ISHIGURO FORMULA

This study examines the intensity (*I*) against recurrence intervals (*Tr*) of 2-year (*p* = 50%), 5-year (*p* = 20%), 10-year (*p* = 10%), 20-year (*p* = 5%), 25-year (*p* = 4%), and 50-year (*p* = 2%) for 1, 2, 3, 4, and 5-hour duration (*t*) [Dorneles *et al.* 2019]. As mentioned before, to calculate constants in Sherman and Ishiguro formulas we need intensity in a given duration (*t*) and recurrence intervals (*Tr*). In this study, the intensity in each *Tr* was calculated using frequency analysis log Pearson type III [Bhat *et al.* 2019]. Table 3 displays the rainfall intensity with varying *Tr* and *t* from log Pearson type III, which was subsequently used to estimate the constants from the Sherman and Ishiguro formulas using Equations (2) and (3). The summary of Sherman and Ishiguro constants is shown in Table 4. The value of constant *a* is directly proportional to the magnitude of the recurrence interval (*Tr*). The value of constant *a* increases with the magnitude of the

Table 3. The rainfall intensity (*I*) for varying recurrence interval (*Tr*) and *t* obtained from log Pearson type III

Duration, *t* (h)	*I* (mm·h^{-1}) at *Tr* (y)					
	2	5	10	20	25	50
1	12.05	22.65	30.90	39.15	42.57	47.39
2	7.04	12.99	17.66	22.24	24.16	26.87
3	4.96	9.72	13.42	17.13	18.66	20.83
4	4.13	8.07	11.44	14.21	15.48	17.27
5	3.80	7.17	9.80	12.42	13.51	15.05

Source: own study.

Table 4. The summary of the constants of the Sherman and Ishiguro

Formula	Equation	Constant	Recurrence interval, *Tr* (y)					
			2	5	10	20	25	50
Sherman	$I = \frac{a}{t^n}$	*a*	11.79	22.06	30.04	38.03	41.34	46.01
		n	0.740	0.721	0.715	0.712	0.711	0.710
Ishiguro	$I = \frac{a}{\sqrt{t} + b}$	*a*	6.51	12.69	17.51	22.31	24.31	27.12
		b	–0.463	–0.438	–0.431	–0.427	–0.426	–0.424

Explanations: *I* = rainfall intensity (mm·h^{-1}), *t* = duration (h).
Source: own study.

recurrence interval (*Tr*), as presented in Table 4. Conversely, constant *b* (for Ishiguro formula) and *n* (for Sherman formula) were inversely proportional to the magnitude of the recurrence interval. Values of *b* and *n* decrease in parallel with the increase in the recurrence interval (*Tr*). The results are consistent with SOEKARNO and ROHMAT [2005], who found the same pattern for the constant value from the SHERMAN and ISHIGURO formulas at varying recurrence intervals (*Tr*). Some researchers attribute values of constants *a*, *b*, and *n* to the influence of meteorological variations and geographical characteristics of a location [MINH NHAT *et al.* 2006]. Thus, further research should be conducted on meteorological fluctuations and geographical factors and their influence on the variability of values of *a*, *b*, and *n*.

Table 5 displays the estimated rainfall intensity from the Sherman and Ishiguro formulas for varying *Tr* and *t*. As presented in Table 5, the intensity shows a decrease with the increase of *Tr* for a given duration. Further, the intensity demonstrates a high value in a shorter duration. The results were concurrent with [DAKHEEL 2017; JAHNVI *et al.* 2014], who confirmed that the high-intensity rainfall occurred for a high recurrence interval and shorter duration.

PERFORMANCE ANALYSIS OF THE EMPIRICAL MODEL, SHERMAN, AND ISHIGURO FORMULA

The rainfall intensity for a given recurrence interval (*Tr*), probability of occurrence (*p*), and duration (*t*) could be calculated based on the empirical model equation. The validation was performed to assess the comparison between rainfall intensity obtained from the empirical model and the formula (Sherman and Ishiguro) with the observed rainfall intensity obtained from the log Pearson type III ($I_{\text{obs-LP}}$). The four statistics measures were employed to assess the comparison results. Figure 8 presents the intensity in various *t* and *Tr* for the observed intensity ($I_{\text{obs-LP}}$), the empirical model, Sherman, and Ishiguro. In Figure 8, the intensity curve shows a negative slope that indicates the intensity tends to decrease as *t* increases for a given *Tr*. In addition, the empirical intensity curve displays a perfect fit with the curve of the observed intensity, which indicates excellent agreement between the estimated intensity from the empirical model with the observed one. The Sherman likewise shows a similar result where the curve of Sherman suits the observed intensity. Conversely, the curve of the Ishiguro relatively deviates from the curve of the observed intensity, which indicates an inaccurate model.

The best fit of the intensity curve from the empirical model and the Sherman formula seems to apply only for the small *Tr* (2, 5, 10, and 20 y), while the curve pattern tends to deviate from the curve of the observed intensity for a high *Tr* (25 and 50 y). Fewer rainfall data have probably become the factor that influences the deviation as BEN-ZVI [2009] described that the accuracy of intensity for a recurrence interval (*Tr*) highly depends on the number of rainfall data.

While referring to probability *p* and recurrence interval (*Tr*), the limited number of high-intensity events due to its small *p* we can see the inaccuracy of the empirical model in estimating intensity with high *Tr* [ALAM *et al.* 2018]. Moreover, the high-intensity tends to show irregular event patterns compared with the low-intensity (which frequently happens); thus, the high-intensity tends to fluctuate. The further explanation of this matter could be addressed by the concept of *p* and *Tr*, where $p = 1/Tr$ [GROUNDS *et al.* 2018].

Thus, rainfall events with high intensity have a smaller *p* (which infrequently happens). Accordingly, it might apply to fewer rainfall events. Limitation of the number of high-intensity probably influences the accuracy of the empirical model, particularly for intensity with a higher *Tr* [VOLPI 2019].

Further, to better assess the model performance, statistical measures comprising the *NSE*, *PBIAS*, index of agreement (*d*), and *RSR* were employed. The results of the model performance are demonstrated in Table 6. As displayed in Table 6, the statistical measures vary between each model for various *Tr*. It was known that the empirical model demonstrates good performance in estimating intensity for all *Tr*, compared with other models, which are indicated by the *PBIAS* values (2.35–3.17), and the *RSR* (0.028–0.031) is smaller than in other models. Furthermore, the *NSE* values ranged from 0.905 to 0.996, which indicated that the model performance was classified as "very good" ($0.75 < NSE \le 1$), and the index of agreement close to 1 (0.986–0.987). This indicates a perfect fit of the model as classified in Table 1. The Ishiguro model seems to be unacceptable for estimating the intensity since its negative values of *NSE*, high *PBIAS*, and *RSR* indicate an unsatisfied model performance for all

Table 5. The estimated intensity (*I*) from the Sherman and Ishiguro formulas for varying recurrence interval (*Tr*) and duration (*t*)

Duration (h)	I (mm·h^{-1}) at *Tr* (y)											
	2		5		10		20		25		50	
	S	I	S	I	S	I	S	I	S	I	S	I
1	11.79	8.88	22.06	10.59	30.04	14.63	38.03	18.68	41.34	20.36	46.01	22.72
2	7.06	5.25	13.38	8.13	18.30	11.23	23.22	14.32	25.26	15.61	28.13	17.42
3	5.23	4.09	9.99	6.85	13.69	9.45	17.39	12.05	18.93	13.13	21.09	14.65
4	4.23	3.46	8.12	6.03	11.15	8.32	14.17	10.60	15.43	11.55	17.20	12.89
5	3.58	3.06	6.91	5.45	9.50	7.51	12.09	9.58	13.16	10.44	14.68	11.64

Explanations: S = Sherman; I = Ishiguro.
Source: own study.

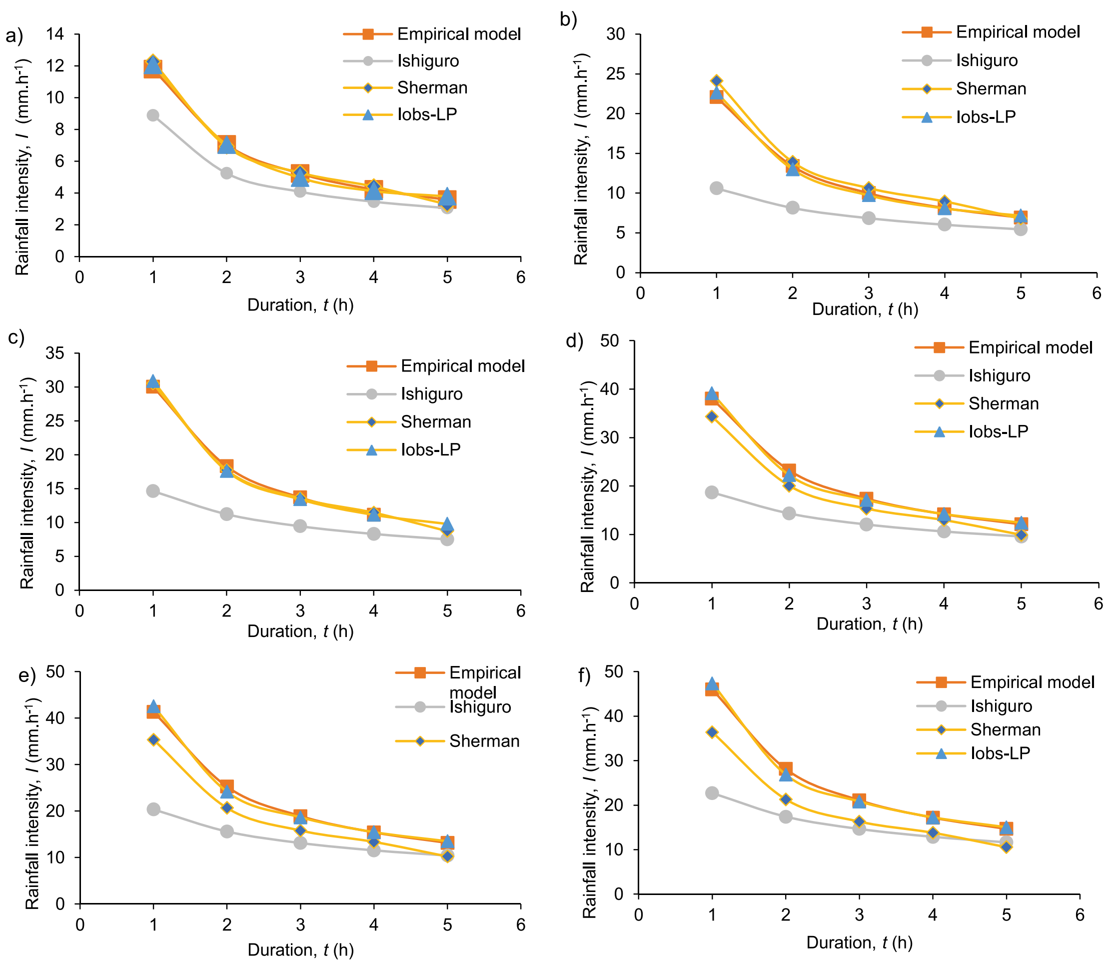


Fig. 8. The intensity versus duration (t) and recurrence interval (Tr) for the observed intensity ($I_{obs\text{-}LP}$), the empirical model, Sherman, and Ishiguro: a) Tr = 2 y, b) Tr = 5 y, c) Tr = 10 y, d) Tr = 20 y, e) Tr = 25 y, f) Tr = 50 y; source: own study

Table 6. The statistics for the performance of the empirical model, models of Sherman, and Ishiguro

Formula for rainfall intensity	Statistical measure	Value at recurrence interval (*Tr*)					
		2 y	5 y	10 y	20 y	25 y	50 y
Empirical model	*PBIAS* (%)	2.34	2.35	2.35	2.35	2.53	3.17
	NSE	0.996	0.991	0.987	0.975	0.938	0.905
	d	0.986	0.987	0.987	0.987	0.987	0.987
	RSR	0.028	0.027	0.029	0.030	0.030	0.031
Ishiguro	*PBIAS* (%)	21.01	33.91	33.43	33.15	33.07	32.98
	NSE	0.67	–0.17	–0.15	–0.14	–0.13	–0.13
	d	0.872	0.759	0.763	0.766	0.767	0.767
	RSR	0.244	0.455	0.448	0.444	0.443	0.442
Sherman	*PBIAS* (%)	6.05	7.85	3.37	12.31	16.91	23.11
	NSE	0.99	0.97	1.00	0.92	0.84	0.70
	d	0.98	0.96	0.99	0.94	0.91	0.87
	RSR	0.04	0.07	0.03	0.12	0.17	0.23

Explanations: *PBIAS*, *NSE*, *d*, *RSR* as in Tab. 1.
Source: own study.

Tr. Conversely, the Sherman acts as a good model in estimating intensity as indicated by the values of *PBIAS*, *NSE*, *d*, and *RSR*, which are in the range of acceptable results. However, the magnitude of the statistical measures, as displayed in Table 6, affirms that the empirical model exhibits better performance than the Sherman.

Further, Table 6 revealed that the statistical performance of all models tends to decrease as *Tr* increases. The empirical model, Sherman, and Ishiguro show a good performance in estimating intensity for small *Tr* (2, 5, 10, and 20 y), whereas the performance quality decreases for high *Tr* (25 and 50 y). This was also confirmed by increasing the *PBIAS*, *RSR*, and lowering *NSE* and *d* values along with increasing *Tr,* as shown in Table 6. Figure 8 affirms this situation where the intensity curve for all models tends to deviate from the observed rainfall intensity as *Tr* increases. The present study found that the rainfall intensity model demonstrates a reliable result mainly for a small *Tr* while less quality prediction for a high *Tr*. The validation results show that the empirical model is the most reliable in estimating intensity for a given *t* and *Tr* in the study area.

CONCLUSIONS

The present study developed an empirical model of rainfall intensity as a function of the probability of occurrence (*p*) and rainfall duration (*t*). The performance of this model was compared with the other formula (Sherman and Ishiguro). The study revealed that the empirical model demonstrates good performance in estimating rainfall intensity compared with the Sherman and Ishiguro formulas, as indicated by the *NSE*, *PBIAS*, *RSR*, and the index of agreement (*d*) values which are rated "very good" in the model performance. Further, the study found that the empirical model exhibits reliable results for the rainfall intensity with a small *Tr* (2, 5, 10, and 20 y) while low estimation for a high *Tr* (25 and 50 y). If associated with rainfall duration (*t*), the reliability of the estimated intensity from the empirical model shows a tendency to decrease as *t* increases. Accordingly, we observe good accuracy mainly for the rainfall intensity with shorter duration (1-, 2-, and 3-hour) while low accuracy for rainfall with long-duration events. Based on the model validation results, it could be affirmed that the empirical model shows good performance, which could be used as a capable and reliable instrument to estimate the rainfall intensity with reasonable accuracy in the study area.

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